The Greenhouse Effect and Carbon Accumulation Dynamics: A General Equilibrium Simulation

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Abstract: A dynamic process on carbon dioxide in the atmosphere, *Y*, is constructed under the general equilibrium framework to examine the property of stationary points of the dynamic process. In this primitive economic model there are two production sectors: agriculture sector and energy sector; and one (aggregate) household. *Y* is externality on food production of agriculture sector. At each time, production and consumption are conducted with *Y* given. The dynamic system of *Y* is subject to two countervailing factors. One is *Y*-raising (negative) factor through the production and consumption of energy by three economic agents at each time. The other is a set of *Y*-reducing (positive) factors through the photosynthetic function of the trees and farm outputs and the repository working of the sea at each time. The externality function of *Y* on the food production, *E*(*Y*), is single-peaked, such as the normal distribution function. The simulation on the dynamic process shows, first, that the stationary points of the process: *Y**; consist of at most 3 points, depending on the positive factor coefficient, which indicates the effect of the photosynthesis on the reduction of *Y*. Second, it shows that there is at least one discontinuous point of the coefficient, whose small deviation creates discontinuous movement of *Y**. It confirms that the environmental tax on the consumption of energy to reduce *Y** might be harmful to the household, depending on the derivative of *E*(*Y*) at *Y**: when *Y* is external economy to food production. Finally, the optimum rate of tax is derived.

Keywords: greenhouse effect; photosynthesis; simulation; general equilibrium theory; environmental tax.

1   INTRODUCTION

The attention with respect to the working of CO_2 has been focused on its *negative* side since the 1980s. Indeed, the argument on the global warming through greenhouse effect gained momentum in the 1980s. Brown [1987] warned the decline of food production due to the greenhouse effect. One of the main concerns of The Earth Summit in Rio de Janeiro in 1992 was whether the participants could agree to adopt the environmental tax on CO_2 emission globally. The agreement was not reached. In 1995 the IPCC concluded that global warming is taking place due to human activity: greenhouse effect. The main aim of the 1997 Kyoto Protocol was to reach agreement for each country to reduce the global warming gases. Stern [2007] still warned the decline of agricultural output due to the greenhouse effect.

It must be noted, however, that there does exist the *positive* side of the working of CO_2: i.e. photosynthesis. In other words, plants cannot grow without CO_2, and

agricultural output is zero or almost zero, when Y, the CO_2 level in the atmosphere, is zero how much inputs such as labor and energy are used. When Y increases from zero, the agricultural output rises with the same inputs.

In this way, if the purpose of the analysis is the one on the overall working of CO_2 on the economy, it may be necessary to assume explicitly the externality function, $E(Y)$, on the agricultural output. First, $E(0)=0$ (or nearly 0) must be assumed. As Y increases, $E(Y)$ rises. Only when Y exceeds a critical level, Y^0, the greenhouse effect may surpass the photosynthesis effect, and agricultural output falls with the same inputs as Y ($>Y^0$) increases. In this paper, this working of CO_2 is explicitly introduced into the production function of agricultural output. For simplicity, $E(Y)$ is assumed to be of the one-peaked function. In other words, the increase of Y provides *external economy* towards the agricultural output when $Y < Y^0$, while it causes *external diseconomy* when $Y > Y^0$.

In this paper, with the explicit introduction of externality function, a dynamic process on Y is constructed under the general equilibrium framework, and the property of the stationary points of the dynamic process is examined. There are two production sectors: agriculture sector and energy sector; and one (aggregate) household in this general equilibrium model. Agriculture sector produces food utilizing labor and energy with Y as external factor, while the energy industry requires only labor to produce energy. Household consumes agricultural output and energy, supplying labor. At each time, production and consumption are made with Y given. This economic model must be constructed in the dynamic system, since the production and consumption of energy raises Y at each time, while there are two mechanisms which reduce Y at each time. One is the photosynthetic function of the trees and agricultural output and so on, and the other is a function of the sea as the greatest repository of carbon. Thus, Y is subject to countervailing factors as time elapses: one is the enhancing factor exhibited by the combustion of fossil fuels through the human activity and the other are reducing factors just mentioned. In this way, this paper develops a primitive economic model to explore the variation of CO_2 through human activity, and examine the property of the stationary points of the dynamic process. A Pigouvian (environmental) tax on energy consumption is also examined.

2   SHORT-RUN GENERAL EQUILIBRIUM MODEL

A primitive general equilibrium (GE) model is constructed, for the purpose of examining the greenhouse effect. By specifying parameters, the short-run GE is derived analytically for use in later simulation.

2.1 General Equilibrium Model and Assumptions

Suppose that there are two firms. The first firm is a farm which produces the crops; Z_f. Whereas the crops is produced by labor: L_1, and energy: H_{f1}, the output depends on Y: CO_2 in the atmosphere. Thus, this farm has the production function, $Z_f=g_1(L_1, H_{f1}, Y)$. The farm determines the production of crops, considering the price of crops: p, wage rate: w, and energy price: p_H. The second firm is the energy industry which produces energy: H_{f2}, using only labor: L_2. It has the production function, $H_{f2}=g_2(L_2)$. The energy industry determines the supply of energy, considering p_H and w. By the profit maximization, the first firm gives rise to the demand for labor: L_1^d, demand for energy: H_{f1}^d, and supply of the crops: Z_f^s. In the same way, the second firm gives rise to the demand for labor: L_2^d, and supply of energy: H_{f2}^s. The demands and supplies depend on the prices: p, p_H, and w.

There is only one (aggregate) household, which consumes the crops: Z_h, and energy: H_h. The (aggregate) household has utility function: $u=u(Z_h, H_h)$ with population N, where u is the utility level felt by the household in consuming the crops and energy. The household determines the optimal consumption of the crops and energy, considering p, p_H, and its income. Following the traditional assumption of general equilibrium theory, it is assumed that the household is the owner of the two firms, so that the profits of the two firms are distributed to the

(aggregate) household. The household income is the sum of profit distribution and wage income: wN. By the household's utility maximization subject to income constraint, it gives rise to the demand for energy: $H_h{}^d$, and the demand for the crops: $Z_h{}^d$. They also depend on the prices: p, p_H, and w.

Given CO_2: Y, the short-run GE is defined by $\{p, p_H, w\}$, which simultaneously satisfies the following three market equilibrium conditions.

$H_{f1}{}^d + H_h{}^d = H_{f2}{}^s$ (the energy market equilibrium)
$Z_h{}^d = Z_f{}^s$,   (the crops market equilibrium)
$L_1{}^d + L_2{}^d = N$ (the labor market equilibrium)

2.2   Specification and Analytical Solution

For the purpose of simulation in later simulation, suppose that production and utility functions are stipulated by the following Cobb-Douglas type.

$g_1(L_1, H_{f1}, Y) = L_1{}^{a1} H_{f1}{}^{a2} E(Y)^{a3}$, $a_1 + a_2 + a_3 \leq 1$ (1)
$g_2(L_2) = L_2$ (2)
$u(Z_h, H_h) = Z_h{}^b H_h{}^{1-b}$, $0 < b < 1$ (3)

The Cobb-Douglas function is one of the most familiar functions, used in economic analysis. The externality function: $E(Y)$, is the new feature in this interdisciplinary work. It is assumed to be of the single-peaked one with Y^0 the single peak with $E(0)=0$. When $Y=0$, the agriculture production is impossible since the photosynthesis is impossible. As Y increases to Y^0, the working of photosynthesis is dominant and the effect on the crops is positive. In economics terminology, Y causes *external economy*. However, as Y surpasses Y^0, the working of greenhouse effect becomes dominant and the effect on the crops becomes negative. In economics terminology, Y causes *external diseconomy*.

In the following sections for simulation, we use two types for this function.

[Type-1]: $E(Y) = \sin(1/(Y/1000 + \pi))$
[Type-2]: $E(Y) =$ normal distribution $(1000, 100)$

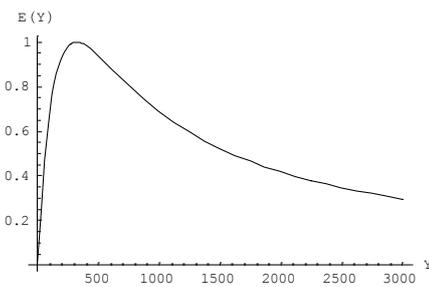The two types of externality function are depicted in Figures 1 and 2.

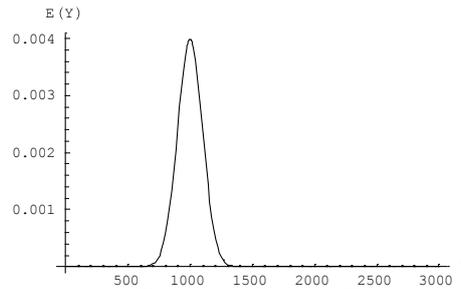
Figure 1. Type-1 Function

Figure 2. Type-2 Function

In the short-run GE, only the relative prices are determined. Under specifications: (1), (2), and (3), given Y, $p_H/p = w/p$ must hold and p_H/p is analytically derived, and utilizing it, *GE quantities*: L_1^*, H_{f1}^*, H_h^*, Z_f^*; are analytically derived as in the following simple form.

$L_1^* = a_1 b N/(1 - b(1 - a_1 - a_2))$
$H_{f1}^* = a_2 b N/(1 - b(1 - a_1 - a_2))$
$H_h^* = (1-b) N/(1 - b(1 - a_1 - a_2))$
$Z_f^* = a_1{}^{a1} a_2{}^{a2} E(Y)^{a3} (bN)^{a1+a2} /(1 - b(1 - a_1 - a_2))^{a1+a2}$

The importance of this derivation consists in the fact that the numerical simulation is quite easily conducted through this derivation..

3   LONG-RUN GENERAL EQUILIBRIUM DYNAMICS

Relaxing the assumption of "fixed Y", we investigate the dynamics of Y, utilizing the *short-run GE quantities* derived in the previous section.

3.1   Formulation of the Dynamics

The analysis in the previous section is called *the short-run general equilibrium model*, since Y is fixed by the assumption. In fact, Y increases through the use of energy in the household's direct consumption and farm's use of energy in the crops production, while it decreases thanks to the absorption by the working of sea and the photosynthetic function of crops. The variation of Y, in turn, causes the variation of the crops output. Thus, the economic analysis of greenhouse effect must be constructed in terms of dynamic system. This dynamic system is called *the long-run general equilibrium dynamics*. In this section, an extension of this sort is attempted.

Formally, as energy is produced, Y increases by the amount of $m_1 \times (H_{f1}{}^*(t) + H_h{}^*(t))$, while Y decreases, first, by the amount of $m_2 \times Y(t)$, thanks to the activity of the sea and, second, $m_3 \times Z_f{}^*(t)$, thanks to the photosynthetic function of crops. Here, for the purpose of later simulation, these workings are linearized for constant parameter $m_i > 0$. (i=1,2,3) In other words, one unit increase of energy consumption causes m_1 increase of Y, etc. Thus, we have a dynamic system

$$dY(t)/dt = F(Y(t)) \equiv m_1 \times (H_{f1}{}^*(t) + H_h{}^*(t)) - m_2 \times Y(t) - m_3 \times Z_f{}^*(t) \qquad (4)$$

where t stands for time.

Note that, except for the $m_2 \times Y(t)$ term, as shown previously only $Z_f{}^*(t)$ depends on $Y(t)$, while $H_{f1}{}^*(t) + H_h{}^*(t)$ is constant. This property is rather robust so long as traditional production functions are utilized. For an instance, if a familiar CES type production function with externality, such as

$$(L_1{}^{a1} + H_{f1}{}^{a1})^{1/(3a1)} E(Y)^{a3}, \ a_1 < 1$$

is assumed instead of (1), the property still holds.

In what follows, the structure of stationary points of (4) is investigated.

3.2 Simulation

The right hand side of (4), $F(Y(t))$, reveals that (4) is globally stable, in the sense that arbitrary initial point, $Y(0)$, converges to some stationary point of (4) since $F(0)>0$ and $F(\infty)<0$. The structure of stationary points of CO_2, $\Psi[P]$, however, is not clear without specifying parameters, where P is the set of parameters such as a_1, a_2, a_3, b, m_1, m_2, m_3, and N.

3.2.1 Type-1 Externality Function Case

Indeed, under type-1 externality function, on the one hand, when

$$a_1 = a_2 = a_3 = 1/4, \ b = 1/2, \ m_1 = 1/1000, \ m_2 = 1/1000, \ m_3 = 10, \ N = 10000000 \qquad (5)$$

there are 3 stationary points of CO_2 levels:

$$\Psi_1[(5)] = \{18.7197, \ 5427.61, \ 6.91778 \times 10^6\}.$$

The graph of $F(Y)$ on the interval [0,10000] is depicted in Figure 3, while the one on [100000,8000000] is depicted in Figure 4. Thus, as is clear from Figure 3, the second stationary point is locally unstable, while the first stationary point is stable. In other words, when CO_2 declines from 5427.61, it declines further since $F(Y)<0$,

while when CO_2 rises from 5427.61, it rises further since $F(Y)>0$. In the same way, as is clear from Figure 4, the third stationary point is locally stable. In other words, when CO_2 declines from $6.91778×10^6$, CO_2 rises back to it since $F(Y)>0$, while when CO_2 rises from this stationary point, CO_2 declines back to it since $F(Y)<0$.
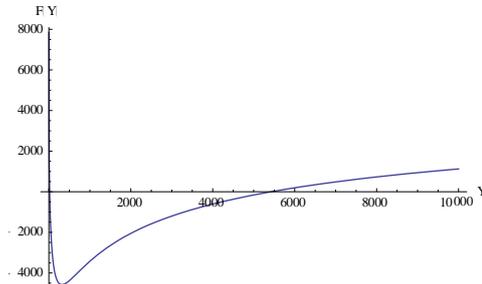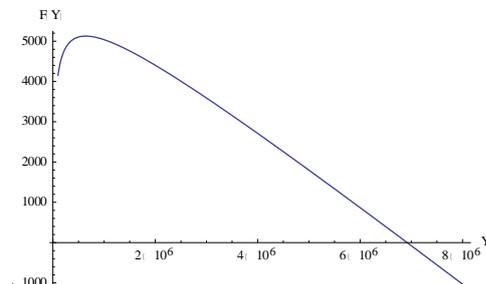

Figure 3. $F(Y)$ on [0,10000]

Figure 4. $F(Y)$ on [100000, 8000000]

On the other hand, under type-1, when

$a_1=a_2=a_3=1/4$, $b=1/2$, $m_1=1$, $m_2=1$, $m_3=10$, $N=10000000$ (6)

there is only one stationary point CO_2 level: $\Psi_1[(6)]=\{8.33142 ×10^6\}$; which is shown to be locally stable.

In this way, we have at most three stationary points of CO_2 levels on the dynamic process, (4).

Next, we conduct an examination of the variation of stationary points by changing only m_3 in (5) with other parameters, a_1, a_2, a_3, b, m_1, m_2, and N fixed at those in (5). Denoting the set of stationary points for parameters in (5) except for m_3 by $\Psi_1[m_3]$, we have the *inverse S-shaped* graph of $\Psi_1[m_3]$: the curve *ABCD* in Figure 5, where the *BCD* part is drawn more clearly in Figure 6.
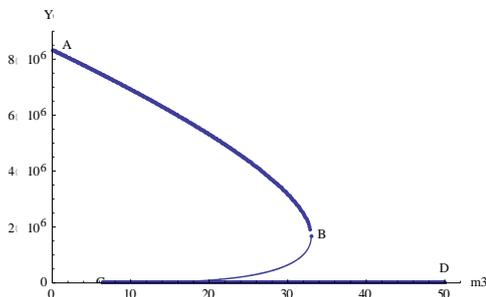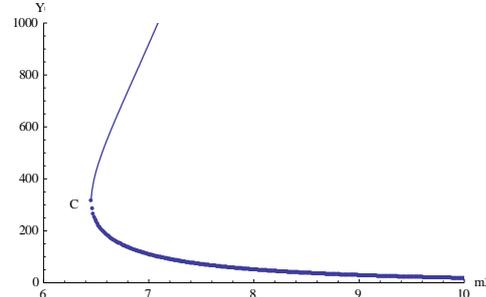

Figure 5. Graph of $\Psi_1[m_3]$ (part1)

Figure 6. Graph of $\Psi_1[m_3]$ (part2)

Note that the *BC* part consists of *unstable* stationary points, where $m_3≈33$ for *B* and $m_3≈6.45$ for *C*. Furthermore from Figure 5 and 6, it is clear that stationary points make discontinuous variation around $m_3≈33$ and $m_3≈6.45$. For example, when m_3 falls from 6.5 to 6.4, the CO_2 in the atmosphere might suddenly increase from the *CD* part to the *AB* part. In the same way, when m_3 rises from 32.5 to 33.5, the CO_2 in the atmosphere might suddenly decrease from the *AB* part to the *CD* part.

3.2.2 Type-2 Externality Function Case

In this subsection, a different externality function is used in order to examine the robustness of the previous subsection. In the simulation, it is assumed that $E[Y]$ is a normal distribution with expected value of 1000 and the standard deviation of 100, as exhibited in Figure 2.

Under type-2 externality function, on the one hand, when (5) is assumed, then there are only one stationary point:

$\Psi_2[(5)] = \{8.333333 \times 10^6\};$

which is globally stable. As in the previous subsection, denoting the set of stationary points for parameters in (5) except for m_3 by $\Psi_2[m_3]$, we can show that

$\Psi_2[100] = \{670.227628, 1329.7819726, 8.333333 \times 10^6\}$

The graph of $F(Y)$ on the interval [0, 2000] is depicted in Figure 7, while the one on [0, 9000000] is depicted in Figure 8.
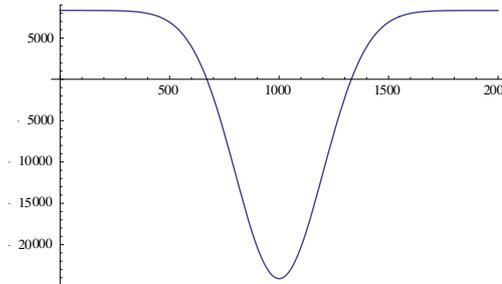

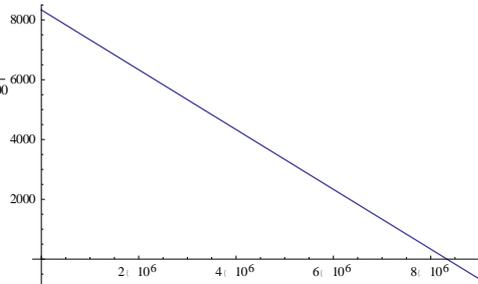**Figure 7.** $F(Y)$ on [0, 2000]

Figure 8. $F(Y)$ on [0, 9000000]

As is clear from Figure 7, the first stationary point is stable, while the second is unstable. In the same way Figure 8 shows that the third stationary point is stable. Note that 8.333333×10^6 in $\Psi_2[100]$ is infinitely close to $25/3 \times 10^6$, but the former is not equal to the latter.

In exactly the same way as in the previous subsection, we conduct an examination of the variation of stationary points by changing only m_3 in (5). Denoting the set of stationary points for parameters in (5) except for m_3 by $\Psi_2[m_3]$, we *do not* have the *inverse S-shaped* graph of $\Psi_1[m_3]$ as in Figures 5 and 6. Instead we have the graph of $\Psi_2[m_3]$ as depicted in Figures 9 and 10. There is a possibility that at extremely large m_3, such point as B in Figure 5 might appear in Figure 9.
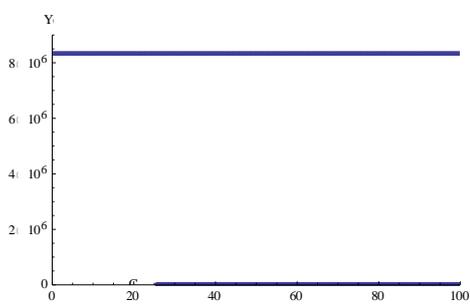

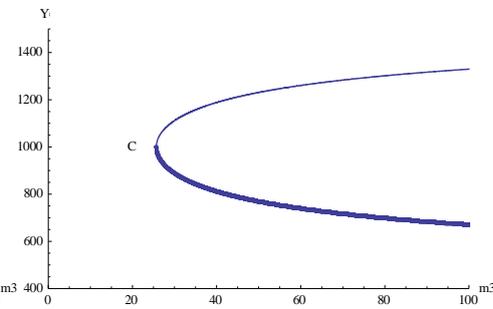**Figure 9.** Graph of $\Psi_2[m_3]$ (part1)

Figure 10. Graph of $\Psi_2[m_3]$ (part2)

Still, there is at least one m_3 with discontinuous variation, as in the previous subsection: when m_3 falls from 26 to 25, the CO_2 in the atmosphere might suddenly increase from, say 1000 to 8.333333×10^6.

4   ENVIRONMENTAL TAX

In the present age of "global warming", in which CO_2 is simple-mindedly regarded as "external diseconomy", one of the economic policies is the taxation on energy consumption to reduce CO_2 in the atmosphere. When the model is constructed in more general situation with *external economy* as well as *external diseconomy* phase, however, it may be worthwhile to examine rigorously the effect of environmental (or Pigouvian) tax on the economy.

In this section with taxation, the primitive general equilibrium model in the previous sections is modified as follows. As above, there are two firms. The first firm is a farm which produces crops, with the production function specified by (1). The second firm is the energy industry which produces energy, with the production

function specified by (2). The modification consists in the imposition of Pigouvian tax on the *consumption of energy*, while the tax revenue is distributed to the household. The tax rate is т>0, and the energy price is $(1+т)p_H$ for the purchaser while it is p_H for the seller. Thus the main modification consists in the one in which the household's behavior is the utility maximization subject to the income constraint with the tax revenues incorporated into the household income.

First, assuming т=1/10, we examine the utility variation through taxation on the *stable* stationary points.

4.1 Utility Variation under Type-1 Externality Function Case

Under type-1 externality function, when parameters are stipulated by (5), the stationary points are given by $\Psi_1[(5)]$, and the corresponding (aggregate) household's utilities: *u*, are given by the following.

{74535.5, 74511.3, 30719.8}

Under type-1 externality function, assuming т=1/10, we have three stationary points of CO_2 on the modified dynamic system with taxation.

{16.3287, 6225.04, 6.75012×10^6}

The corresponding household's utilities are derived as in what follows.

{73313.6, 73285.9, 30799.3}

Thus, environmental tax reduces the *stable* stationary CO_2 level. Note, however, that the utility level falls by the taxation for the first stationary CO_2 level, since CO_2 operates as the *external economy*, while the utility level rises by the taxation for the third stationary CO_2 level, since CO_2 operates as the *external diseconomy.* Essentially the same conclusion holds under type-2 externality function.

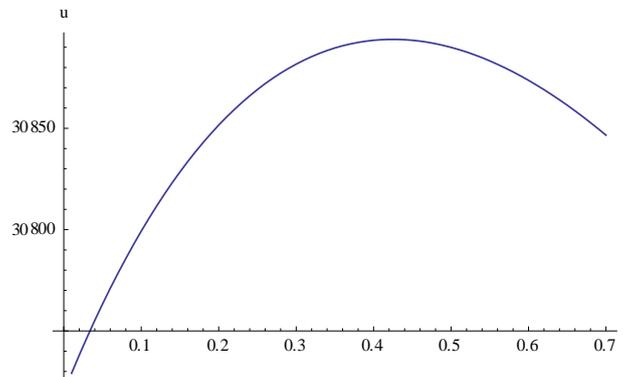### 4.2 The Optimum Rate of Environmental Taxation for the External Diseconomy Case

Figure 11. Various Tax Rates and Utilities

In this subsection, the optimal taxation is examined for the third stationary point in $\Psi_1[(5)]$. In general equilibrium analysis, the firms are owned by the (aggregate) household, so that the optimality is examined by considering the (aggregate) household's utility level. As т rises from zero, the corresponding household's utility for each stationary point rises as in Figure 11, reaching the maximum value, 30893.9, at т=0.43. When т rises further, the household's utility level falls. The optimal rate for the environmental taxation, in this simulation, is 43%.

5   CONCLUSIONS

In the present age, people's interest appears to be focused on the global warming and food shortage due to the greenhouse effect: the negative side of carbon dioxide, CO_2. There does exist, however, the positive side of CO_2, known as the photosynthesis thanks to CO_2. When there exists small amount of CO_2 in the atmosphere, the effect of positive side on the crops production would dominate the one of negative side, while the effect would be reversed when there exists large amount of CO_2. The aim of this paper was to fully examine the dynamic process of CO_2 variation by assuming the working of CO_2 as the externality to the crops production. CO_2 in the atmosphere expands through the human activity, so that in this paper the general equilibrium (GE) theory was utilized to examine the dynamic process.

After the introductory remarks in section 1, in section 2, assuming two-industry, one-(aggregate) household GE model, we analytically derived the short-run GE: the crops output, energy consumption etc; given CO_2 in the atmosphere. Since the energy consumption raises CO_2 in the atmosphere, in section 3, the dynamic process of CO_2 variation was formulated as a differential equation. After the global stability was confirmed, it was shown, furthermore, by simulation approach that small variation of photosynthesis effect might cause drastic variation of the stationary CO_2 in the atmosphere. This discontinuity was confirmed robust by utilizing two types of externality function. The single-peaked externality function guaranteed the existence of at least one discontinuous stationary point, so that if the multiple-peaked externality function is observed and assumed, there might exist a lot of discontinuous points. Finally, in section 4 it was asserted that the environmental tax must be applied with sufficient care, by the confirmation that it might be harmful to the society if it is applied without knowing where we are on the externality function. It was also shown that the computation of optimal tax rate is possible so long as sufficient information is provided. With the addition of the applicability to the optimal environmental taxation, it was shown that the introduction of photosynthesis effect, as well as the greenhouse effect, into the economic model, adds richness to the abstract model.

ACKNOWLEDGMENTS

REFERENCES

Brown, L.R., *State of the World*, W.W. Norton & Company, 1987.
Fukiharu, T., The Greenhouse effect and carbon accumulation dynamics:
   a general equilibrium simulation, 2011.
   (http://www.cc.aoyama.ac.jp/~fukito/greenhouse&agri1.pdf)
Pigou, A.C., *The Economics of Welfare*, London, Macmillan, 1920.
Stern, N., *The Economics of Climate Change: The Stern Review*, Cambridge University Press, 2007.