

Sustainable Watershed Management by Fuzzy Game Optimization

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Abstract: An untraditional multi-objective programming approach, called as fuzzy game multi-objective optimization model (FGMOM), is illustrated to discuss suitability and significances in the problems of sustainable watershed management. FGMOM is developed based on the combination of fuzzy set theory, game theory and traditional multi-objective programming. The model is implemented to support decision making process for balancing economic and environmental paradox in reservoir watershed management. Particularly in this study, FGMOM is used as an alternative tool for analyzing strategic interaction between economic development and environmental protection which is involving vague and imprecise information related to data, model formulation, and the decision maker's preferences. Results show that analysis of economic and environmental balance can be easily interpreted to aid the decision maker for watershed management. For comparison, a traditional multi-objective approach was used to contrast the comprehensive use and robustness from FGMOM analysis. In addition, the application of FGMOM is further discussed.

Keywords: Fuzzy mathematical programming; Watershed management; Multi-objective optimization; Game theory; Conflict analysis.

1. INTRODUCTION

Multi-objective programming problems exist almost everywhere, including environmental facilities [Giannikos, 1998], water resources management [Lee and Wen, 1996], watershed management [Lee and Wen, 1995]. In these problems, one of the challenges for the decision makers (DMs) is to choose an appropriate alternative among a set of alternative actions. In the past decade, many techniques have expanded based on multi-objective programming approaches, such as artificial neural network [Wen and Lee, 1998], fuzzy set theory [Wang and Liang, 2004], game theory [Nishizaki and Sakawa, 2001], genetic algorithm [Leung and Wang, 2000], grey theory [Hsu and Wen, 2000]. In this study, we developed a fuzzy game multi-objective optimization model (FGMOM) for economic and environmental balance in a reservoir watershed.

For the past three decades, there have been lots of studies in literature discussing the topic of fuzzy games. Butnariu [1978] perhaps first introduced the concept of fuzzy sets in non-cooperative games. Aubin [1981] discussed cooperative games with fuzzy coalitions. Nishizaki and Sakawa [1995] developed equilibrium solutions of fuzzy goals for games in fuzzy and multi-objective environments. Dhingra and Rao [1995] combined cooperative game theory and fuzzy set theory to yield new cooperative fuzzy games, and further examined the utility and applicability in an engineering design process. Nishizaki and Sakawa [2000] formulated a fuzzy cooperative game based on a linear programming problem with fuzzy parameters for a production problem. Molina and Tejada [2002] analyzed lexicographical solutions for fuzzy games, and studied their properties and characterization. Li and Cheng [2002] developed fuzzy multi-objective optimization methods for fuzzy constrained matrix games. Branzei et al. [2003] studied the cone of convex cooperative fuzzy games and examined their fuzzy Shapley value. Garagic and

Cruz, Jr. [2003] first defined a new approach to N -person static fuzzy non-cooperative games and developed a solution concept. Branzei et al. [2004] introduced an egalitarian solution for convex fuzzy games by adjusting the classical Dutta–Ray algorithm for convex crisp games. Bector et al. [2004] also proposed fuzzy linear programming models to solve fuzzy matrix games. Peldschus and Zavadskas [2005] combined fuzzy sets and matrix game theories for multi-criteria decision-making. They illustrated a case study for selecting the variants water supply systems. Hwang [2007] introduced two types of “reduced game” to cooperative fuzzy games and defined the related properties of consistency and converse consistency. Larbani [2009] surveyed most of the approaches for solving non-cooperative fuzzy games in normal form. In addition, applications of these games were also discussed. Yu and Zhang [2010] proposed a generalized form of fuzzy game and extended as a cooperative fuzzy game. They also provided a practical application for production problem. Chen et al. [2010] formulated a game framework for strategic behavior of supply chain partners based on fuzzy multi-objective programming.

To our best knowledge, combined implementation of FGMOM has never been performed in watershed management in that one needs to balance between economical development (EcoD) and environmental protection (EnvP). Consequently, this study was undertaken using a fuzzy game approach to provide the DMs a clear choice in selecting an appropriate alternative. Results of FGMOM for watershed management may further expand the application of traditional multi-objective optimization in water resources area. For comparison, a traditional multi-objective approach was used to contrast the comprehensive use and robustness from FGMOM analysis.

2. MODELS

A traditional model of multi-objective programming is

$$\begin{aligned} \text{Max } Z(x) &= [Z_1(x), Z_2(x), \dots, Z_p(x)] \\ \text{s.t. } g_j(x) &\leq 0, j = 1, 2, \dots, m \\ x_k &\geq 0, k = 1, 2, \dots, n \end{aligned} \quad (1)$$

Solution of the multi-objective programming model employs the concept of Pareto optimal, also known as the non-inferior solution. The bi-objective for EnvP is to minimize impact on the environment (less total phosphorus (TP) load), whereas EcoD is to maximize the utility function of income. Thus, the bi-objective programming model is

$$\begin{aligned} \text{Min } EnvP &= Z_1(x) \\ \text{Max } EcoD &= Z_2(x) \\ \text{s.t. } g_j(x) &\leq 0, j = 1, 2, \dots, m \\ x_k &\geq 0, k = 1, 2, \dots, n \end{aligned} \quad (2)$$

Considering the imprecise nature inherent in human judgments in multi-objective programming problems, the DM can specify his/her preferences for the corresponding fuzzy goals. A fuzzy goal can be quantified by eliciting a corresponding membership function. After the membership functions $\mu_1(Z_1(x))$ and $\mu_2(Z_2(x))$ have been elicited from the DM, the fuzzy bi-objective programming model can be defined as

$$\begin{aligned} \text{Max } [\mu_1(Z_1(x)), \mu_2(Z_2(x))] \\ \text{s.t. } g_j(x) &\leq 0, j = 1, 2, \dots, m \\ \mu_1(Z_1(x)) &= h_1(x) \\ \mu_2(Z_2(x)) &= h_2(x) \\ x_k &\geq 0, k = 1, 2, \dots, n \end{aligned} \quad (3)$$

By introducing a general aggregation function, we can define a general fuzzy multi-objective decision making problem by

$$\begin{aligned} & \text{Max } \mu_D(\mu(Z(x))) & (4) \\ & \text{s.t. } g_j(x) \leq 0, j = 1, 2, \dots, m \\ & \quad \mu_1(Z_1(x)) = h_1(x) \\ & \quad \mu_2(Z_2(x)) = h_2(x) \\ & \quad x_k \geq 0, k = 1, 2, \dots, n \end{aligned}$$

Value of $\mu_D(\mu(Z(x)))$ can be interpreted to represent the overall degree of satisfaction with the DM's multiple fuzzy goals.

Fuzzy decision developed by Bellman and Zadeh [1970] is presented as

$$\mu_D(\mu(Z(x))) = \text{Min} [\mu_1(Z_1(x)), \mu_2(Z_2(x))] \quad (5)$$

Combining Equations (4) and (5), we can interpret the fuzzy decision making problem as

$$\begin{aligned} & \text{Max Min } [\mu_1(Z_1(x)), \mu_2(Z_2(x))] & (6) \\ & \text{s.t. } g_j(x) \leq 0, j = 1, 2, \dots, m \\ & \quad \mu_1(Z_1(x)) = h_1(x) \\ & \quad \mu_2(Z_2(x)) = h_2(x) \\ & \quad x_k \geq 0, k = 1, 2, \dots, n \end{aligned}$$

or equivalently

$$\begin{aligned} & \text{Max } v & (7) \\ & \text{s.t. } v \leq \mu_1(Z_1(x)), \\ & \quad v \leq \mu_2(Z_2(x)), \\ & \quad g_j(x) \leq 0, j = 1, 2, \dots, m \\ & \quad \mu_1(Z_1(x)) = h_1(x) \\ & \quad \mu_2(Z_2(x)) = h_2(x) \\ & \quad x_k \geq 0, k = 1, 2, \dots, n \end{aligned}$$

If all of the membership functions $\mu_i(Z_i(x))$ are linear, the max-min problem becomes a linear programming problem. Hence, one can obtain an optimal solution by applying the simplex method of linear programming using LINDO software package [Schrange, 1991]. In this study, two players represent EnvP and EcoD, respectively. Their payoffs will be identified as the following process.

Initially, each player will not be satisfied with the results from the other player's objective, i.e., FGMOM result for player 1 is far above his/her objective value (i.e., more TP load than what he/she expects), and the corresponding value for player 2 is way below his/her goal (i.e., much less than he/she wants). Thus, two players will begin another round of negotiation and concession. For example, during the second negotiation, each player will then set his/her objective value very close to initial $EnvP_{goal}$ and $EcoD_{goal}$ and see what happens. As would be expected, each player is still not satisfied after the second negotiated results. Consequently, a series of concessions and negotiations will be conducted by adjusting player's objective values, i.e., player 1 would relax more about his/her environmental concern while player 2 lowers the economic income. After further negotiations, difference between the reset objective values and FGMOM results becomes less divergent. The negotiation process continues until final solutions of $EnvP^*$, $EcoD^*$ can be identified as:

For player 1:

$$EnvP^* \leq EnvP_{goal} \quad (8)$$

And for play 2:

$$EcoD^* \geq EcoD_{goal} \quad (9)$$

Pairs of solution ($EnvP^*$, $EcoD^*$) are called as Nash equilibrium points.

In summary, the computing process of FGMOM is briefly listed as

- Step 0: establishing original multi-objective optimization model;
- Step 1: setting payoffs as strategies of players;
- Step 2: fuzzifying objectives by membership functions;
- Step 3: making decision by Bellman and Zadeh's approach;
- Step 4: checking Nash equilibrium (if not, go back to Step 1);
- Step 5: obtaining optimization solutions.

3. CASE STUDY

Developments for various land uses in reservoir watersheds are important in that from a viewpoint of economic development, one of the important things is to allocate the resources to different users for maximizing the income as higher as possible. On the other hand, for watershed management, one of the important policies is to allocate the distribution of land uses in which pollution would not reduce the assimilative capacity in their receiving water-bodies, nor affecting water quality in reservoirs. Agricultural activities and forest are major types of land uses in the mountainous area. However, residential and recreational lands are not only highly profitable, but also result in polluted runoff. They should be set their maximum areas for environmental protection and soil conservation.

Non-point source pollution (also known as diffuse pollution) originates from natural (forest) and agricultural land development (e.g., plantation of tea, betel nut, mustard), feedlot (free-range chicken farms) as well as community and recreational sites. In this study, the land uses are classified into major four categories (Table 1).

For each land use type, unit quantity of pollutant (for EnvP) and economic income (for EcoD) must be known. Since phosphorus is the controlling element for eutrophication, only TP is used in this study. Export TP values shown in Table 1 were taken from the comprehensive study conducted by Wen et al. [2005]. Income values from any economic development associated with respective land use were adopted from Council of Agriculture, Government of Taiwan [2009]. Two objective functions and constraint set in our study are listed in Table 2.

A membership function for TP impact is shown in Fig. 1(a). The desired TP quantity is 14,200 kg/yr and its mapping membership grade is 1.0. Similarly, the maximum possible quantity is 26,800 kg/yr and its grade is assigned as zero. Another membership function for total income in the entire watershed is shown in Fig. 1(b). The maximum income is 394 million USD/yr and its mapping membership grade is 1.0. The minimum possible income may be 223 million USD/yr and its grade is assigned as zero.

4. RESULTS AND DISCUSSION

4.1 FGMOM for identifying the goals of EnvP and EcoD

FGMOM can be converted into crisp (e.g., non-fuzzy) linear programming models that are solved with conventional algorithm using LINDO software package [Schrange, 1991]. The trade-off processes of watershed-management optimization are first established. Consequently, four types of candidate solutions are generated and showed in Table 3. Type I indicates the solutions of two objectives as $Z_1 = 19,150$ kg TP/yr and $Z_2 = 327$ million USD/yr based on the payoffs of 0, 0.3, 0.5 for both of $\mu_1(Z_1(x))$ and $\mu_2(Z_2(x))$. Type II will have the payoff values as $\mu_1(Z_1(x)) = 0.7$ and $\mu_2(Z_2(x)) = 0, 0.3, \text{ and } 0.5$, respectively. The solutions of two objectives are $Z_1 = 17,980$ kg TP/yr and $Z_2 = 320$ million USD/yr. However, Type III have the payoff values as $\mu_2(Z_2(x)) = 0.7$ and $\mu_1(Z_1(x)) = 0 \text{ and } 0.3$, respectively. The solutions are $Z_1 = 20,800$ kg TP/yr and $Z_2 = 343$ million USD/yr. Type IV has an extreme payoff of $\mu_1(Z_1(x)) = 1.0$ and $\mu_2(Z_2(x)) = 0$. The solutions are $Z_1 = 14,200$ kg TP/yr and $Z_2 = 224$ million USD/yr.

Results are summarized in Figure 2 for better illustration. The Nash equilibrium points of Types I-III range from 17,980 to 20,800 kg TP/yr for EnvP; and that of objective 2 (EcoD) between 320 and 343 million USD/yr. Area of decision variable residential (x_1) is between 745 and 1,879 ha for its high income. Forest land (x_2) is between 41,957 and 43,088 ha for

its lower TP export load. Agricultural activities (x_3) have their maximum area (2,166 ha) due to its lower TP export load and higher income. Reservation (non-planted land, water body and grass land area, x_4) is 4,000 ha.

Table 1. Parameter values in the objective for reservoir watershed-management.

Parameter	Residential/recreational areas (x_1)	Forest (x_2)	Agricultural activities (x_3)	Reservation (x_4)
Export coefficient of TP (kg/ha/yr)	2.8	0.3	1.0	0.2
Income of land use or development ($\times 10^3$ USD/ha/yr)	30	6	16	0

Table 2. Multi-objective model for a case study of the Tseng-Wen Reservoir.

Model component	Function
Objectives:	
1. TP loads	Min $Z_1(x) = 2.8x_1 + 0.3x_2 + 1.0x_3 + 0.2x_4$
2. Total income	Max $Z_2(x) = 30x_1 + 6x_2 + 16x_3 + 0x_4$
Constraints:	
1. Land availability	$x_1 + x_2 + x_3 + x_4 = 50,000$
2. Forest area	$x_2 \geq 35,778$
3. Soil property	$x_3 \leq 2166$
4. Minimum agricultural area	$x_3 \geq 164$
5. Minimum residential/recreational areas	$x_1 \geq 180$
6. Land slope	$x_1 + x_3 \leq 4995$
7. Minimum water-body/river-bank areas	$x_4 \geq 4000$
8. TP assimilative capacity	$2.8x_1 + 0.3x_2 + 1.0x_3 + 0.2x_4 \leq 63,500$
9. Non-negativity	$x_1, x_2, x_3, x_4 \geq 0$

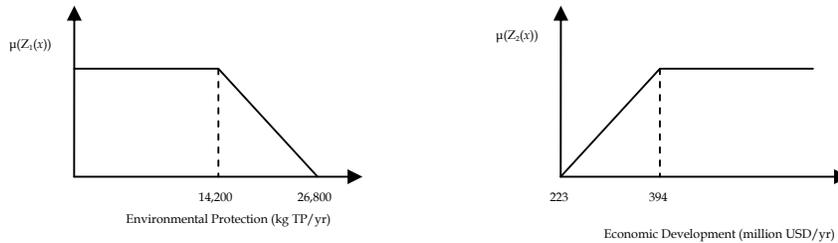


Figure 1. Membership functions for two objectives. (a) TP load and (b) income.

Table 3. Results of FGMOM for a case study of the Tseng-Wen Reservoir.

Payoffs ($\mu(Z_1(x)), \mu(Z_2(x))$)	Z_1 (kg TP/yr)	Z_2 (million USD/yr)	x_1 (ha)	x_2 (ha)	x_3 (ha)	x_4 (ha)
(0, 0), (0, 0.3), (0, 0.5) (0.3, 0), (0.3, 0.3), (0.3, 0.5) (0.5, 0), (0.5, 0.3), (0.5, 0.5)	19150	327	1214	42620	2166	4000
(0.7, 0), (0.7, 0.3), (0.7, 0.5)	17980	320	745	43088	2166	4000
(0, 0.7), (0.3, 0.7)	20800	343	1879	41957	2166	4000

(1, 0)	14200	224	180	36008	164	13648
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4.2 Decision Making: Nash equilibrium versus Pareto optimality

Traditional multi-objective programming methods can generate many candidate solutions, expanding a wide range of the objectives. Results are possibly difficult for the DMs to select which alternative. For example, the DMs of economic affairs may choose the one with larger income; on the other hand, the people in environmental field may choose the lower TP load. To realize the usefulness of FGMOM model, a traditional multi-objective analysis, constraint method, was also performed for comparison. Detailed procedures of this method are shown elsewhere [Lee and Wen, 1996]. The solutions of constraint method could be Pareto optimality [Messac and Mattson, 2004]. Therefore, analysts would then select several appropriate values falling within the pay-off-table range and subject to constraint of another objective function. Results of multi-objective analysis are also shown in Figure 2. However, there are a lot of alternatives for the watershed-management problem (depending on the values the analysts selected). For environmentalist, he/she tends to select alternatives with lower TP loading (lower left points in Figure 2). Conversely, for economists, the choice is apparently the ones with higher economic income (upper right points in Figure 2). Results of the multi-objective method can only be utilized for the DMs without any prejudice about environment versus economy.

On the other hand, FGMOM is one of aggregative approaches and construct limited satisfactory solutions according to the DMs' preferences. The DMs served as players who need to negotiate each other and have a series of concessions. The model has a range of the solutions but converge efficiently to Nash equilibrium. Objective values will converge until the Nash equilibrium is achieved (Figure 2). Range of Nash equilibrium serves a base for the DMs to select the choice they wish to proceed. Since the range is not too wide, the DMs may feel comfortable to make decision. Thus the eventual decision can accommodate both environmental concerns and economic development.

In short, constraint method provides a set of data that analysts identify many solutions. The solutions are all Pareto optimality, so the quality of solutions is better than that of FGMOM. But they are possibly difficult for the DMs to select one. FGMOM indicates Nash equilibrium points that players have focused their goals. After several negotiation attempts, Nash equilibrium points were obtained. It is easy for the DMs to decide the one balancing EnvP and EcoD.

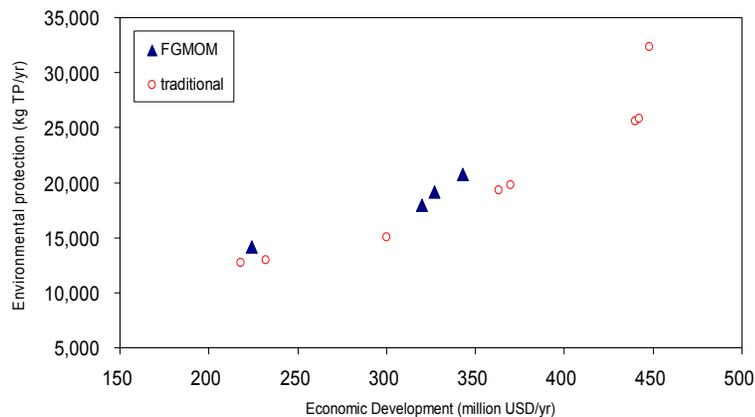


Figure 2 Results of FGMOM and traditional multi-objective analyses.

5. CONCLUSIONS

A framework for decision making of land uses in reservoir watershed-management is performed. Further applications along with fuzzy theory, game theory and multi-objective programming to deal with conflict analysis for balancing environmental protection and economic development are also formulated and resolved. The developed models have been

applied to an illustration for identifying reservoir watershed-management solutions. Modeling results have been generated, demonstrating complex tradeoffs among the balance between economic development and environmental protection. Solutions, based on FGMOM, reflect compromises between maximized income and minimized TP impact in a watershed. The results suggest that these models are also applicable to many other environmental management problems.

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