LPV Approximation of Distributed Parameter Systems in Environmental Modeling

G. Belforte [a], **F. Dabbene** [b], **P. Gay** [c]

[a]Dipartimento di Automatica e Informatica - Politecnico di Torino
corso Duca degli Abruzzi 24 - 10129 Torino - Italy
belforte@polito.it

[b]IRITI-CNR - Politecnico di Torino
corso Duca degli Abruzzi 24 - 10129 Torino - Italy
dabbene@polito.it

[c]Dipartimento di Economia e Ingegneria Agraria Forestale e Ambientale
Università degli Studi di Torino
via Leonardo da Vinci, 44 - 10129 Grugliasco (To) - Italy
gaypaolo@agraria.unito.it

Abstract: Environmental systems often involve phenomena that are continuous functions not only of time, but also of other independent variables, such as space coordinates. Typical examples are transportation phenomena of mass or energy, such as heat transmission and/or exchange, humidity diffusion or concentration distributions. These systems are intrinsically distributed parameter systems whose description usually requires the introduction of partial differential equations (PDE). Therefore, their modeling can be quite complex, both for what concerns the model construction and its identification. Indeed, a typical approach for the simulation of such systems is the use of finite element techniques. However, this kind of description usually involves a huge number of parameters and requires time-consuming computation while not being suited for identification. For this reason, such models are generally not suitable for control purposes. In many cases, however, the involved phenomena depend on the independent (space) variables in a smooth way, and for fixed values of the independent variables, input/output relations can be satisfactorily represented by linear invariant models. In such conditions, a possible alternative to PDE consists in representing the physical system with a Linear Parameter Varying (LPV) model whose parameters are functions of the independent variables. The advantage of this approach is the relatively simple model obtained, which is directly suitable for control purposes and can be easily identified from input/output data by means of classical techniques. Moreover, optimal identification schemes can be derived for such models, allowing the optimization of the number of measurements. This can be particularly useful in several environmental applications for which the cost of measurements represents a severe constraint. In this paper, the derivation of LPV models for the representation of distributed phenomena in environmental systems is discussed and illustrated with a simulated and a practical example.

Keywords: Distributed parameter systems, Approximation, LPV models.

1 INTRODUCTION

In environmental and agricultural sciences complex systems need often to be described with mathematical models. The development of model structures adequate for practical use is carried on with different approaches, depending on the goals of the modeling process as well as on the available information. Therefore, at one extreme, large simulation models based on the best available knowledge on the involved processes are developed. Such models are often too complex to be satisfactorily identified from experimental data and too detailed to be suitable for control needs. At the other extreme, simple low order input-output "black box" models are derived only processing available data with no or little use of *a priori* knowledge on the involved process. The choice is therefore between highly detailed models, that reduce the uncertainty in the system representation to a minimum, and "simple" models suitable for control purposes that however imply higher uncertainty in the system representation. Drawbacks of, and alternatives to such approaches are nicely discussed in a recent paper by Young and Chotai [2001].

It is well accepted that models to be used in control applications need to be simple and robust to uncertainties, nevertheless the possibility to incorporate some knowledge on the involved process in the model construction is a desirable feature. Also, the possibility to state a correspondence between the model behavior, its parameters and the physical nature of the real system is often seen as a great advantage.

Most environmental and agricultural processes are intrinsically distributed parameter systems, and their behavior should therefore be described using partial differential equations (PDE) that, besides being function of time, depend also on spatial coordinates. Possible examples are given by processes in which mass or energy transport phenomena occur. The resulting models are infinite dimensional state models that, in general, are difficult to be identified and managed even in the simple linear case.

Indeed, applying finite-differences techniques it is possible to approximate such models by equivalent finite state models driven by ordinary differential equations in which the infinite number of states is replaced by a "large number of states". Such models are usually presented in discrete-time form to be implemented on digital computers and their large number of states increases for increasing required accuracy in approximating the original infinite dimensional system. The resulting model is therefore suitable for simulations but is far too complex for control purposes, although it has moderate uncertainty.

In control engineering practice high dimensional systems are commonly approximated with lower order linear time-invariant (LTI) models. The case of distributed parameter systems regarded as systems with a "large number of states" is not an exception. Hence, low order approximating models can be derived for distributed parameter systems. It is also possible to add some physical interpretation to the different parameters and features of the approximated model. For example, some of its time constants could be regarded as diffusion constants or transport time constants between the input and the output of the distributed parameter real system. Simplified time-invariant linear parameter models however lose any information about the spatial structure of the original system and cannot account for it although they can be satisfactory from an input-output point of view.

In practice, there are cases in which some information about the spatial structure of the system should be maintained in the simplified model so that results derived for particular settings with respect to the spatial positions of inputs and outputs can be used also when different input-output conditions are of interest.

Hence, some simplified model structure that still preserves some information about the spatial structure of the system is needed. Such structure can be provided by Linear Parameter Varying (LPV) models consisting of a linear lumped parameter model in which the parameters are not constant, but are functions of an extra, possibly vector valued, variable x that can be regarded as an input determining the "operating condition" of the model [Rugh and Shamma [2000], Leith and Leithead [2000], Shamma and Xiong [1999]]. By selecting the x operating condition to be the spatial coordinate of the inputs and/or outputs of the system some information about the spatial structure is included in the simplified model and can be used to derive results valid throughout the system volume.

Remark that this kind of model remains basically a simplified input-output model. It is therefore not suited for replacing the infinite dimensional model driven by partial derivative equations (or its finite element approximation) when simulating the internal behavior of the distributed parameter system.

In this paper, we discuss the use of LPV models for describing distributed parameter systems when information about the spatial structure of the system is needed. In Section 2, the notation and the main ideas of this approach are developed. In Section 3, a simulated example relative to the determination of the temperature in a bar plunged at its extreme in two fluids at different temperatures is reported. In Section 4, the real case of soil cooling after forced steam heating during a disinfestation process is discussed and the proposed techniques are applied to real data. Finally in Section 5 some conclusions are drawn.

2 LPV APPROXIMATION OF DISTRIBUTED PARAMETER SYSTEMS

Consider a distributed parameter system subject to an input signal u. We are interested in the prediction of the forced response of a distributed variable $y(t;x)$ to the input signal $u(t)$ for any time t and spatial coordinate x. To this extent we introduce am approximating model which is constituted by the cascade of a delay and an LPV model. Both the delay and the LPV parameters are function of x. For the study of such LPV systems with parameter-dependent delays, both for analysis and control, the interested reader is referred to Wu and Grigoriadis [2001]. Moreover continuous time representations are in general replaced with discrete time models.

The resulting model is reported in Figure 1. It follows that the delay block, taking into account transport delays in the process, is defined as follows

$$\tilde{u}(k) = q^{-\Delta(x)}u(k) \tag{1}$$

where k is (discrete) time, $\Delta(x)$ is an unknown function to be estimated and q^{-1} is the usual unit-delay operator.

The LPV model is based on an autoregressive dynamic structure with exogenous input (ARX)

$$A(q^{-1};x)y(k;x) = B(q^{-1};x)\tilde{u}(k) + e(k) \ , \tag{2}$$

where $e(k)$ takes into account measurement and modeling errors and the regressors $A(q^{-1};x)$ and $B(q^{-1};x)$ are defined as

$$\begin{aligned} A(q^{-1};x) &= 1 + a_1(x)q^{-1} + a_2(x)q^{-2} + \dots \\ &\quad + a_{na}(x)q^{-na} \\ B(q^{-1};x) &= b_0(x) + b_1(x)q^{-1} + b_2(x)q^{-2} + \dots \\ &\quad + b_{nb}(x)q^{-nb} \ . \end{aligned} \tag{3}$$

This model is referred to as ARX(na, nb). The parameters $a_i(x)$, $i = 1, \dots, na$, and $b_i(x)$, $i = 0, \dots, nb$, are unknown (continuous) functions of the parameter x and are to be estimated.

A two step identification approach can then be performed.

First: A set of n different values of x, $\{ \ x_1, \ \ x_2, \ \ \dots \ \ x_n \ \}$ is chosen. Forcing x to assume each one of the n values, n input-output sets of data are collected. Consequently, n different LTI ARX models with the same structure are identified.

Second: The $a_i(x)$, $i = 1, \dots, na$, and $b_i(x)$, $i = 0, \dots, nb$, parameter functions of the LPV model are derived interpolating the corresponding parameters of the n LTI models. Suitable interpolation can be performed using splines or polynomials.

With the resulting LPV model forecast and control of the output $y(k;x)$ can then be performed for any value of x, even different from those used in the identification stage.

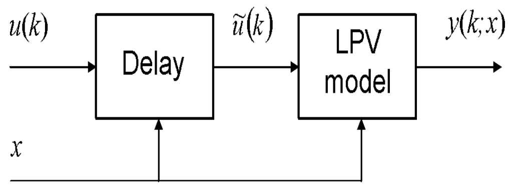

Figure 1: LPV approximating model for single input distributed parameter systems .

3 LPV APPROXIMATION: A SIMULATED CASE

In this section we present the simple problem of non-steady conduction in one space dimension. We consider the case of a one dimensional aluminium bar of length $l = 100cm$ plunged at its extremes into two separated fluids at different temperatures. We assume the temperature $y(t;x)$, $x \in [0,l]$, to be the output and the temperature $T_{F1}(t)$ of the first fluid to be the input. The temperature $T_{F2}(t)$ of the second fluid is assumed to be constant.