Dynamic System Analysis and Environmental Problems

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Abstract: Instability analysis of dynamic systems has become popular since Krugman and Venables [1995] showed that as the transportation technology improves two nations are divided into the wealthy north and the poor south via small shock and they finally converge to each other by the further improvement of transportation. In their instability approach in terms of simulation, the transportation cost, *T*, is the crucial parameter. Thus, when *T* is large symmetric equilibrium of two nations is stable. As *T* declines through innovations, symmetric equilibrium becomes instable; the specialization of the two nations. In this paper, it is shown that this approach has similarity to environmental problems. First, it is shown that depending on the scale of the economy (parameter), biomass may be extinct by commercial fishery, where general equilibrium model in Smith [1974] is utilized. Incidentally, Smith's result is extended in the sense that we have smaller biomass (stock concept) than social optimum, rather than larger catch (flow concept), which is expected by Smith. Second, introducing pollution into the traditional two-sector growth model, where the production of industrial sector reduces the endowment of working hours in this economy, it is shown that the economy may collapse depending on the severity of pollution. Stable property of neoclassical school disappears in this case, too. Remedial policy is also considered. It is shown that how the tax revenue from the imposition of tax on the polluter is distributed is important for the policy to be effective.

Keywords: development, instability, biomass, pollution, simulation.

1. INTRODUCTION

Stability analysis of the general equilibrium in the 1960s has made new development in the 1990s. In the 60s, the main interest was to investigate whether the competitive general equilibrium system under decreasing returns to scale in every sector is stable. In spite of economists' expectation, the conclusion was rather negative. In the 90s, a new light was shed by Krugman and Venables [1995]. Accepting the instability property of economic system, they examined what can be explained under the instability. The research may be classified rather as a comparative statics. Parameter in their model is transportation cost, *T*, between the two countries. When *T* is large symmetric equilibrium of two countries is stable. As *T* declines through the innovation, symmetric equilibrium becomes instable. Instability in this case implies the specialization of one country to manufacture and the one of the other to agriculture. The former country is the "north", a rich country, with high real wage, while the latter is the "south", a poor country, with low real wage. As *T* declines further, approaching to one, however, the two countries become similar, in the sense that the real wages tend to be equal. Their analysis has another distinction from the one in the 60s. They heavily utilize simulations due to the complicated structure of the model. This paper starts with the extension of their analysis. It proceeds to examine other topics from environmental problems.

2. World History under Constant Returns and Increasing Returns: Comparison

Krugman and Venables [1995] constructed a two-country, two-good general equilibrium model with Dixit-Stiglitz type monopolistic competition in manufacture sector, where *M* is the output of the manufacture good and *A* is the output of agriculture sector. Households are assumed to maximize utility U given income Y, where utility function is of Cobb-Douglass type: $U= M^{\mu} A^{1-\mu}$. M is the index of n kinds of differentiated manufacture goods, given by CES type function. The price of manufacture good, G, is the hedonic price computed from the cost minimization.

Manufacture goods are produced from labor and intermediate goods, while the intermediate goods are produced from manufacture goods. In this way, a part of the manufacture goods are consumed by the households, and the rest are consumed by the firms as the intermediate goods. For the purpose of simplification, α of total cost is assumed to be spent on the purchase of intermediate goods, $0<\alpha<1$. Since in the long run profit is zero, the expenditure of households and domestic firms, E_i is given by

$$E_1= \mu Y_1+ \alpha w_1 \lambda_1/(1-\alpha),$$
$$E_2= \mu Y_2+ \alpha w_2 \lambda_2/(1-\alpha) \qquad (1)$$

where w_i is the wage rate for the manufacture sector and λ_i is the share of the manufacture sector in the total labor force for the ith country. Agriculture sector uses only labor as input. Assuming the production function of the agriculture sector to be $A(1-\lambda_i)= (1-\lambda_i)^v$, the national income of the ith country, Y_i, is given as

$$Y_1=w_1 \lambda_1+A(1-\lambda_1),$$
$$Y_2=w_2 \lambda_2+A(1-\lambda_2) \qquad (2)$$

When v=1, it is the case of constant returns to scale for the agriculture sector, assumed by Krugman and Venables [1995].

Transportation cost T is introduced in the trade equilibrium conditions. Denoting the price of the manufacture good of the ith country by G_i, and $\sigma=1/(1-\rho)$, G_1 is determined in equilibrium by

$$G_1^{1-\sigma}=\lambda_1 w_1^{1-\sigma(1-\alpha)} G_1^{-\alpha\sigma}+ \lambda_2 w_2^{1-\sigma(1-\alpha)} G_2^{-\alpha\sigma} T^{1-\sigma} \qquad (3)$$

and G_2 is determined in equilibrium by

$$G_2^{1-\sigma}=\lambda_1 w_1^{1-\sigma(1-\alpha)} G_1^{-\alpha\sigma} T^{1-\sigma}+ \lambda_2 w_2^{1-\sigma(1-\alpha)} G_2^{-\alpha\sigma} \qquad (4)$$

The firms in manufacture sector are assumed to be under monopolistic competition, so that in the long run profit is zero. The zero profit condition for the manufacture sector of the two countries requires the following wage equations for the two countries;

$$(w_1^{1-\alpha} G_1^{\alpha})^{\sigma}/(1-\alpha)=E_1 G_1^{\sigma-1} + E_2 G_2^{\sigma-1} T^{1-\sigma} \qquad (5)$$
$$(w_2^{1-\alpha} G_2^{\alpha})^{\sigma}/(1-\alpha)=E_1 G_1^{\sigma-1} T^{1-\sigma}+ E_2 G_2^{\sigma-1} \qquad (6)$$

In the short run, given λ_1 and λ_2, from (1), (2), (3), (4), (5), (6), variables G_1, G_2, E_1, E_2, Y_1, Y_2, w_1 and w_2 are determined. It is called the short run equilibrium.

Under this short run equilibrium there may be difference of wages between manufacture and agriculture sectors in each country. If there is a difference, labor force moves from the lower wage sector to the higher one. This adjustment process is expressed by the following differential equations, where w_i^A is the wage rate for the agriculture sector of the ith country, dx/dt is the differentiation of x with respect to time, t

$$d\lambda_1/dt=w_1-w_1^A$$
$$d\lambda_2/dt=w_2-w_2^A \qquad (7)$$

It must be noted that the international movement of labor is not allowed in Krugman and Venables [1995].

Long run equilibrium for each T is defined by $d\lambda_i/dt=0$ (i=1,2) .

When the production function of the agriculture sector is assumed to be $A(1-\lambda_i)= (1-\lambda_i)^v$, it is easy to show that $\lambda_1=\lambda_2=\mu$ is the symmetric equilibrium and equilibrium values are given by

$$\lambda=\mu,\ w=(1-\mu)^{v-1},\ Y=(1-\mu)^{v-1},$$
$$G^{1-\sigma(1-\alpha)}= \mu(1-\mu)^{(v-1)(1-\sigma(1-\alpha))}[1+T^{1-\sigma}],$$
$$E=(1-\mu)^{\mu}/(1-\alpha) \qquad (8)$$

whose subscripts; 1 and 2, are deleted for simplicity. If v=1, (8) is the same as (14.18) in Fujita, Krugman, and Venables [2000, p.249]. In order to analyze dynamic system small deviations of variables from the above symmetric equilibrium; $d\lambda$, dw, dE, and dG, must be derived. If we define

$$Z=[1-T^{1-\sigma}]/[1+T^{1-\sigma}]=\{\mu(1-\mu)^{(v-1)(1-\sigma(1-\alpha))}[1-T^{1-\sigma}]\}/G^{1-\sigma(1-\alpha)} \qquad (9)$$

the small deviations of variables must satisfy

$$[(1-\sigma)+\alpha\sigma Z]\frac{dG}{G}=\frac{Z}{\mu}d\lambda+$$
$$[1-\sigma(1-\alpha)]Z/(1-\mu)^{v-1}dw \qquad (10)$$

$$\sigma\, dw+(1-\mu)^{v-1}\left[\frac{\alpha\sigma-(\sigma-1)Z}{1-\alpha}\right]\frac{dG}{G}=\frac{Z}{\mu}dE \qquad (11)$$

$$\frac{dE}{\mu}=\left[\mu+\frac{\alpha}{1-\alpha}\right]dw+(1-\mu)^{v-1}$$
$$\left[(1-v)\mu+\frac{\alpha}{1-\alpha}\right]\frac{d\lambda}{\mu} \qquad (12)$$

(9), (10), (11), and (12) correspond with (14.20), (14.21), (14.22), and (14.24) in Fujita, Krugman, and Venables [2000, pp.250-1], respectively. Indeed, if v=1, the same expression is obtained. When v=1 and profit maximization is assumed, $w_1^A= w_2^A=1$. Under $\sigma=5$, $\alpha=0.4$, $\mu=0.55$, Krugman and Venables[1995] made a simulation on the dynamic system (7), and obtained Figure 1 and Figure2.

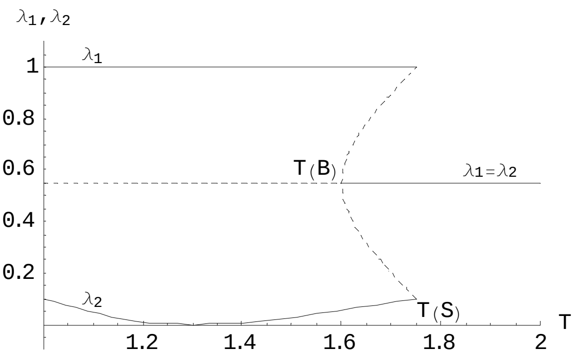

Figure 1: Bifurcation when v=1

In Figure 1, the solid curve implies stable equilibria, dashed curve implies the unstable equilibria, and T(B) is the break point, while T(S) is the sustain point. The break point implies the value of *T*, such that for given $T \geqq$ T(B), the symmetric equilibrium, depicted as the curve with $\lambda_1=\lambda_2$, is stable on (7), while for given *T* $\leqq$T(B), the symmetric equilibrium is unstable on (7). When v=1, there exists T(S), such that for given *T*>T(S) there is no specialization equilibrium, depicted as the curve with λ_1 or λ_2, on (7) as shown in Figure 1. Figure 2 shows the real wages for each country on the equilibria. Thus, in Figure 2, as *T* decreases until T(B), real wages rise mildly on the symmetric equilibria.

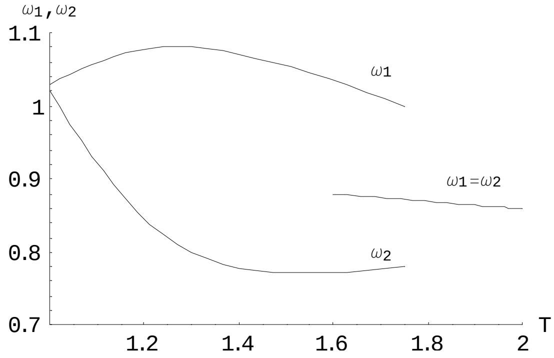

Figure 2: Real Wages when v=1

However, if *T* is smaller than T(B) symmetric equilibrium is unstable on (7), thus specialization equilibrium may emerge. On this specialization equilibrium one country's real wage, ω_1, rises sharply, while the other country's real wage, ω_2, falls sharply. The former is the north and the latter is the south. However, as *T* decreases further, the difference between those real wages narrows.

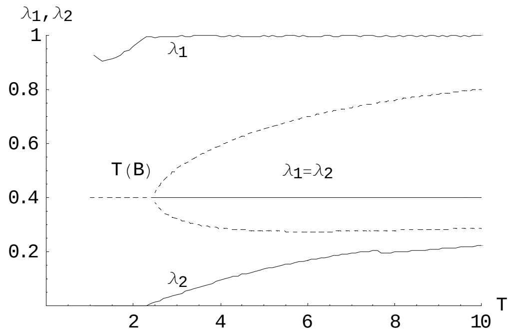

Figure 3: Bifurcation when v=2

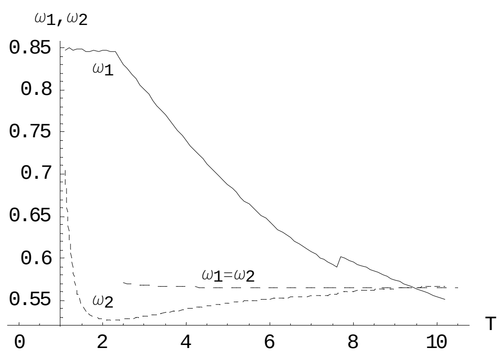

Figure 4: Real Wages when v=2

When v>1, profit maximization cannot be assumed, and we assume that average product is paid to workers in agriculture sector, following Ethier [1982]. Under v=2, $\sigma=5$, $\alpha=0.5$, μ

=0.4, a simulation is conducted on the dynamic system (7). Figure 3 and Figure 4 correspond with Figure 1 and Figure 2, respectively. Except for the non-existence of T(S), they are rather similar.
It can be shown that as v increases, T(B) increases. (See my home page)

3. Biomass and Its Extinction

In this section, we examine a topic, the common property replenishable natural resources; *e.g.* ocean fishery, hunting ground, or ground water supply, following Smith [1974]. Suppose that there are two consumption goods; q_1 is amount of the first good, say, fish, produced from a common property natural resource, say, biomass X, and labor, N_1, while q_2 is amount of "normal good", say rice, produced from land and labor N_2. Thus, we have production functions

$$q_1=f^1(N_1, X)$$
$$q_2=f^2(N_2)$$

It is assumed that identical fishing firms (firm 1) and identical rice-producing firms (firm 2) exist on a closed interval $[0,n]$. Each firm type maximizes profit given p: price of fish, and w: wage rate. Rice is numeraire good, and rice price is normalized at 1. Identical households exist on the closed interval $[0,n]$. Each household has the same utility function, $u(q_1, q_2)$, maximizing utility subject to income restraint. For simplicity, it is assumed that each household possesses all the shares of firm 1 and firm 2. $\underline{N}$ is the endowment working hours possessed by each household. Short-run general equilibrium is derived by adding labor market equilibrium condition, given X:

$$N_1+N_2=\underline{N}$$

This model is dynamic in essence. For, at the general equilibrium, while fish are caught by the amount of $n\ q_1$ from biomass of fish X, X itself grows. Let $F(X)$ be growth rate of the biomass of fish which satisfies

$$F(0)=F(\underline{X})=0,\ F'(\hat{Y})=0, \text{ and } F''(X)<0\ \ \forall X\geqq 0 \qquad (13)$$

for some $0<\hat{Y}<\underline{X}$.
The dynamic system for biomass is defined by

$$dX/dt= F(X) -n\, f^1(N_1, X) \qquad (14)$$

When $dX/dt=0$, it is called bionomic equilibrium.

The main aim of Smith [1974] was to make graphic exposition of production equilibrium transformation (production possibility frontier) and bionomic equilibrium transformation. The production equilibrium transformation is defined by

$$P(\underline{N},X)=\{\{q_1,\ q_2\}|\ q_1= f^1(N_1,\ X),\ q_2=f^2(N_2),\ N_1+N_2=\underline{N}\ \}$$

It was shown that the curve is downward sloping with usual assumptions on f^1 and f^2. The bionomic equilibrium transformation in Smith [1974] is defined by $B(N)=B^*\cup B^{**}$ where

$$B^*=\{\{q_1, q_2\}|\ q_1= f^1(N_1, X),\ q_2=f^2(N_2),\ N_1+N_2=\underline{N}\ ,\ F(X)=nf^1(N_1, X),\ 0\leqq X<\hat{Y}\ \}$$

$$B^{**}=\{\{q_1, q_2\}|\ q_1= f^1(N_1, X),\ q_2=f^2(N_2),\ N_1+N_2=\underline{N}\ ,\ F(X)=nf^1(N_1, X),\ \hat{Y}\leqq X<\underline{X}\}$$

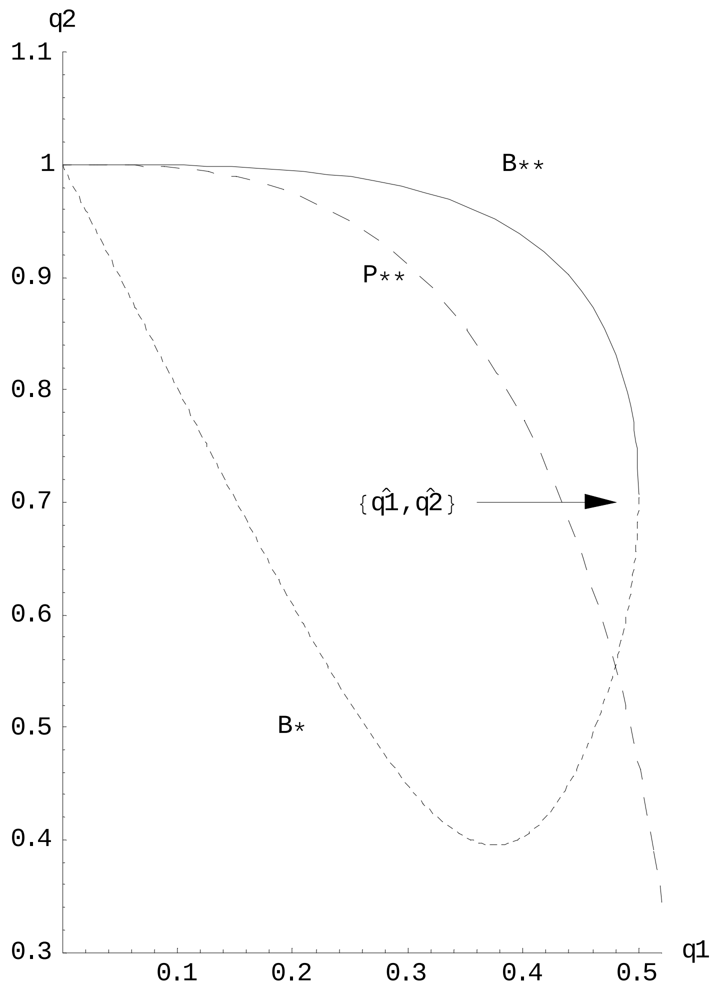

Figure5: Bionomic Equilibrium

If we assume the functions to be of Cobb-Douglass type;

$$f^1(N_1, X)=N_1^{\alpha} X^{1-\alpha},$$
$$f^2(N_2)= N_2^{\beta}, \qquad (15)$$
$$u(q_1, q_2)=q_1^{\gamma} q_2^{1-\gamma},$$
$$F(X)=X(1-2\delta X)$$

we may depict B* and B** as in Figure 5 where $\alpha=1/3$, $\beta=1/2$, $\delta=1/2$, and $\{\hat{q1}, \hat{q2}\}$ corresponds to the case: $X=\hat{Y}=\delta/2$.
Smith [1974] attempts to check whether the competitive bionomic equilibrium is characterized by a larger fishing effort and a larger catch than social optimum, with the conclusion that "this need not be the case unless more restrictive assumptions are imposed on the production function and utility function" (p.114). Rather, it was shown that the competitive bionomic equilibrium biomass, X^*, is smaller than the social optimum biomass, X^{**}, in Fukiharu [1990, p.18]. This conclusion depends on Smith's result. Smith [1974] proved that X^{**} belongs to B**. Fukiharu [1990, p.18] proved furthermore that B** is concave to the origin as shown in Figure 5. Furthermore, he showed that competitive production frontier, say, P**, corresponding to X^* crosses the bionomic equilibrium transformation from below, as shown in Figure 5. In this situation, $X^*<X^{**}$ holds.
Next, the possibility of extinction of the biomass is examined. If the functions are specified as in (15), then the dynamic system for biomass, (14), is stipulated by

$$dX/dt=-X^2+2\delta X-n\nu^{\alpha}\underline{N}^{\alpha}X^{1-\alpha}$$
where $0<\nu=\alpha\gamma/\{\alpha\gamma+\beta(1-\gamma)\}<1$

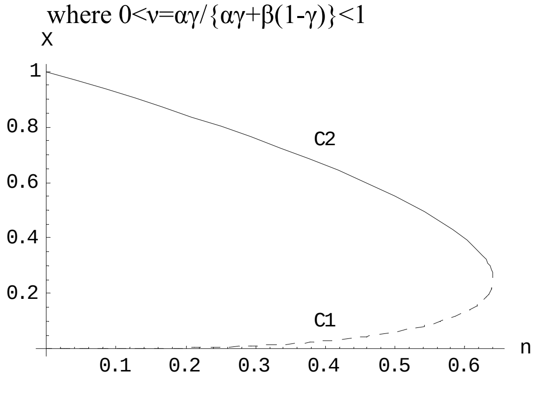

Figure6: Bionomic Bifurcation

This dynamic system has similarity with (7). When n is large, for any initial value $X(0)$, $X(t)$ converges to 0: extinction of biomass. As n declines, there arises the possibility of stable biomass equilibrium. In Figure 6, dashed curve, C1, is the locus of unstable equilibrium, while solid curve, C2, is the locus of stable equilibrium. Figure 6 is depicted for $\alpha=1/3$, $\beta=1/2$, $\gamma=1/2$, $\delta=1/2$, and $\underline{N}=1$. For example, if $n=0.6$ and $X(0)=0.2$, $X(t)$ converges to $X^*= 0.4$. However, if $X(0)=0.1$, $X(t)$ converges to 0.
The policy of tax on fish is shown to be effective in preventing the extinction of biomass.

4. Neoclassical Growth Model with Pollution

Following the traditional two-sector growth model, suppose that there are a consumption-good producing sector (the first sector) and a capital-good producing sector (the second sector) where both sectors use labor and capital for production. The traditional two-sector growth model assumes that the labor force (working hours) of a society grows at the annual rate of ξ. Under additional assumptions (*e.g.* the first sector being more capital intensive than the second) the process of capital accumulation is globally stable. We examine in this section what would happen if the second sector causes pollution, reducing the endowment working hours by the amount proportional to its output where the proportional coefficient is given by μ. For N_i: labor input and K_i: capital input, production functions are given by the Cobb-Douglass type

$$g^1(N_1, K_1)=N_1^{\alpha} K_1^{1-\alpha}$$
$$g^2(N_2, K_2)=N_2^{\beta} K_2^{1-\beta}$$

Total labor hours at time t, $N(t)$, changes by the following differential equation.

$$dN(t)/dt=\xi N(t)-\mu g^2(N_2, K_2)$$

Under assumption of classical saving function: only capitalists save, dynamic system of $k=K/N$ is given by

$$dk/dt=(1-\alpha)^{\beta}(1-\beta)^{1-\beta}(k^{1-\beta}+\mu k^{2-\beta}) - \xi k \qquad (16)$$

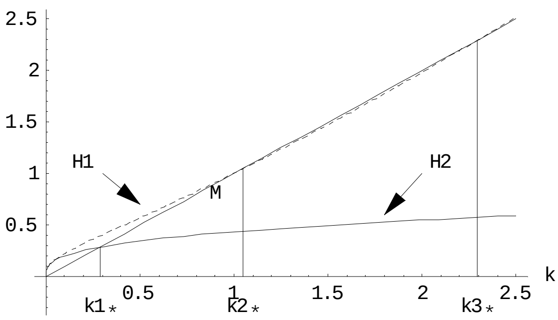

Figure 7: Instability by Pollution

When $\mu=0$, stable equilibrium is given by k1* in Figure 7, where M corresponds with ξk in (16), while H2 corresponds with $(1-\alpha)^{\beta}(1-\beta)^{1-\beta}k^{1-\beta}$ in (16). This is a standard result. When $\mu>0$, however, there arises the possibility of instability, depending on μ. As shown in Figure 7 where $\alpha=1/3$, $\beta=1/2$, $\xi=1$, and $\mu=1.3$, H2 is replaced by H1 and there are two equilibrium points; k2*= 1.0494 and k3*= 2.29398. In this equilibria, k3* is unstable, while k2* is stable. It is possible to depict bifurcation for this case.
It is also possible to show that the policy of tax on polluter with tax receipt payment on workers is effective in preventing the instability, while tax receipt payment on capitalists is ineffective in preventing the instability.

5. Conclusion

This paper started with the extension of Krugman and Venables [1995]. They assumed monopolistic competition with increasing returns for the manufacture sector, while assuming decreasing returns for the agriculture sector. The latter assumption follows the tradition of economics since Malthus [1798]. As is well-known, the US agriculture has shown the increasing returns due to technological innovations since her Independence. In Section 2, the assumption of increasing returns was made for the agriculture sector, and similar conclusion was derived: as transportation cost, *T*, declines through innovations, symmetric equilibrium becomes instable and specialization may emerge. In Section 3 of this paper, an other application to environmental problems was examined. Smith [1974] constructed a simple general equilibrium model with biomass. The main purpose in this model was to compare the competitive (commercial) fishing equilibrium with optimal fishing. Smith could not achieve clear conclusion regarding whether the former is larger than the latter, as is expected. In this section, it was shown that biomass in the commercial fishing is smaller than the one in the optimal fishing. Next, specifying the production and utility functions by Cobb Douglass type, comparative statics was conducted. It was shown that as the scale of the economy (*n*: parameter) rises, the possibility of extinction of biomass by commercial fishery rises. In Section 4, introducing pollution into the traditional two-sector growth model, where the production of industrial sector reduces the working hours in this economy, it was shown that the economy may collapse depending on the severity of pollution (μ : parameter). Stable property of neoclassical school disappears in this case, too. Remedial policy was also considered. It was shown that how the tax revenue from the imposition of tax on the polluter is distributed is important for the policy to be effective.

6. References

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