An Approach for the Aggregation of Albedo in Calculating the Radiative Fluxes over Heterogeneous Surfaces in Atmospheric Models

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Abstract: In the atmospheric models, the underlying surface consists of patches of solid and liquid parts and different plant communities, creating very heterogeneous picture in the grid-cell. In those cases, numerical modelers usually make a simple average to determine albedo as the grid-cell average albedo, a key variable in the parameterization of the land-surface radiative transfer over it. This method leads to uncertainties in the parameterization of the boundary layer processes when heterogeneities exist over the grid-cell. Also, a physics-based analysis indicates that there is a significant departure of the albedo above such a heterogeneous surface from that calculated by simple averaging, seriously affecting the calculated values of momentum, heat and water vapour transfer from the surface fluxes into the atmosphere. The object of this paper is to consider the assumptions for aggregating the albedo over a very heterogeneous surface, and, then propose a method for evaluating a general expression for it. The suggested expression for the albedo is compared with the conventional approach, using a common parameterization of albedo over the grid-cell, and the two-patches grid-cell with a simple geometrical distribution and different heights of its components.

Keywords: Aggregation of surface parameters; Albedo; Modelling

1. INTRODUCTION

Common situation in the environmental modeling is that the underlying surface consists of patches of solid and liquid parts and different plant communities, resulting in rather heterogeneous picture in the grid-cell as well as in calculating the fluxes of momentum, energy, carbon-dioxide, and moisture between the underlying surface and the atmosphere [Sellers, 1997; Yang et al., 2001]. In those cases numerical modelers usually make a simple (arithmetical) average [Wetzel and Boone, 1995; Jacobson, 1999] to determine albedo as the grid-cell-average albedo, a key variable in the parameterization of the land surface radiation and energy budgets. However, a physics-based analysis indicates that there is a significant deviation of the albedo above such a heterogeneous surface from that calculated by simple averaging, seriously affecting the calculated values of quantities describing surface biophysical processes like land surface energy budgets, canopy photosynthesis and transpiration, urban area physics and snow melt, among others. It is therefore important to understand the general behavior and limitations of approaches for aggregating the albedo over heterogeneous grid-cell in the form in which they are currently used in land surface models.

With these issues in mind, this paper considers a new approach for aggregating the albedo over a very heterogeneous surface in land surface schemes for use in environmental models. We start by discussing the basic assumptions of our approach for aggregating the albedo, and, then propose a method for evaluating a general expression for it. The suggested expression for the albedo is compared with the conventional approach, using a common parameterization of albedo over the grid-cell, and the two-patches grid-cell with a simple geometrical distribution and different heights of its components. This is presented in Section 2. The results are summarized in the concluding section.

2. PHYSICAL BASIS OF AGGREGATION

In this Section we will propose a general procedure for aggregating the albedo over a very heterogeneous surface, more precisely, for accounting the effect of different height levels and nature of the surfaces, which are parts of a given grid-cell. We demonstrate this cumbersome procedure by using a rather simplified model i.e. two-patch grid-cell with a simple geometrical distribution and different heights of its components. After discussing the basic assumptions of our approach, we derive a general expression for the aggregated albedo. The derived expression for the albedo of the particularly designed grid-cell is compared with the conventional approach, using a common parameterization of albedo over the same grid-cell as used by Delage et al. [1999].

First of all let us state the basic assumptions. We suppose that the basic constituent of the albedo, coming from the grid-cell, describes the diffuse, homogeneous single scattering of incoming radiation from a given surface. This simplifying assumption neglects the multiple scattering effect and the dependence of the albedo on zenithal angle of the incident radiation. Apparently, within this approach, the geometry plays an essential role. In our approach, a part of radiation reflected from the lower surface is completely absorbed by the lateral sides of the surface lying on a higher level. Consequently, the idea is to calculate the ratio of the reflected energy lost in this manner by calculating the solid angle in which these lateral sides are seen from each point of the lower surface. Let us note that we assume that observer is sufficiently high, so the height of the observer need not be taken into account as in the work of Schferdtfeger [2002].

To calculate the radiant energy flux *dE/dt* we introduce the total intensity of radiation I obtained from the monochromatic intensity by integrating it in the range of the whole spectrum. Taking into account that within our approach I is a constant, we can write down our basic expression following Liou [1980]

$$\left(\frac{dE}{dt}\right) = I\ dS\ \cos\vartheta\ d\Omega \tag{1}$$

where, dS is the infinitesimal element of surface on which radiation comes or reflects from, $cos\theta$ describes the direction of the radiation stream, while $d\Omega = sin\theta\ d\theta\ d\varphi$ is the element of solid angle within which our differential amount of energy is confined to.

We shall now explain our analytic treatment for the most general case considering the average albedo of the properly chosen grid-cell of the area S as presented in Fig. 1. For the simplicity we assume that this region consists of two surface types, with different albedos and heights. Accordingly, we assume that this grid-cell is divided into two subregions having the areas S_1 and S_2 with corresponding albedos α_1 and α_2, respectively, while relative height of higher surface is h. In order to define the position of a particular point we have to use global (x, y, z) reference frame as well as the local reference frame (x', y', z') assigned to each point with axes

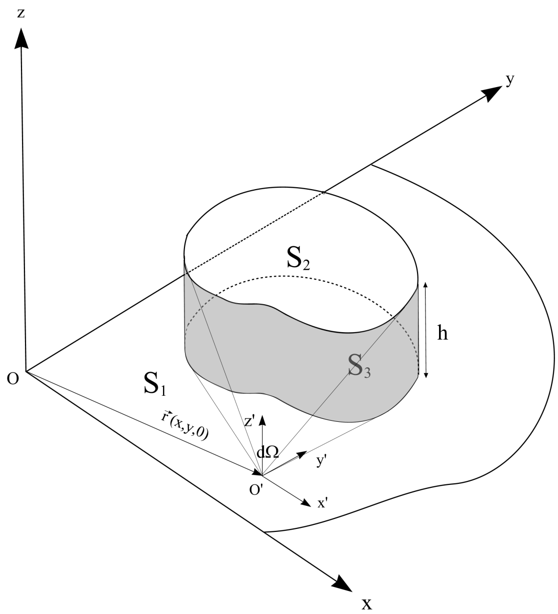

Figure 1. Schematic representation of the grid-cell of an arbitrary geometry consisting of two surfaces of the relative height *h* and of the differential solid angle used in definition of $(dE/dt)_l$. Notation follows the text.

parallel to the corresponding global ones, which is used for the calculation of the solid angle under which the vertical boundary between two surfaces is seen from the given point (Fig. 1).

According to the conventional approach, the average albedo $\overline{\alpha}_c$ over the grid-cell of an arbitrary geometry is given as

$$\overline{\alpha}_c = \alpha_1 \sigma_1 + \alpha_2 \ \sigma_2 \qquad (2)$$

in terms of the fractional covers $\sigma_i = S_i/S \ (i = 1,2)$ where $S = S_1 + S_2$ is the total grid-cell area.

Our idea is to introduce the "loss coefficient" k $(0 < k \leq 1)$, which measures relative radiant flux lost from the reflected beam from the lower surface due to their nonzero relative height. We must emphasize that our basic assumption is that the flux of radiation which reaches the vertical boundary surface of the area S_3 (which lies in the plane normal to the surfaces S_1 and S_2) is completely lost. This means that we are not taking into account the contribution of the radiation reflected from the surface S_3 to the total reflected flux of radiation. In that way we calculate the average albedo $\overline{\alpha}_n$ of this same grid-cell as

$$\overline{\alpha}_n = (1-k) \ \alpha_1 \sigma_1 + \alpha_2 \ \sigma_2 \ . \qquad (3)$$

One way of accounting the possible reflection from the vertical boundary would be to add the term including the albedo of vertical boundary which, however, need not be equal to α_2 at all, which poses an additional problem.

Finally, our definition of loss coefficient brings us to the following relation

$$k = \frac{\left(\dfrac{dE}{dt}\right)_l}{\left(\dfrac{dE}{dt}\right)_h} \qquad (4)$$

where $(dE/dt)_h = IS\pi$ is amount of flux which land surface of area S_1 emits into the upper half-space [Liou, 1980], while $(dE/dt)_l$ is the part of the total energy coming from surface S_1 towards surface S_3.

The amount of emitted flux reaching the vertical boundary is sought as a sum of all infinitesimal amounts of radiant flux emitted from the infinitesimal surface element $dxdy$ (centered around the point with position vector $\vec{r}$), confined in the solid angle $d\Omega$ under which the element dxdy "sees" the surface S_3. (Fig. 1). The boundaries for the integration over azimuthal (φ_l, φ_u) and zenithal (θ_l, θ_u) angles in terms of the global coordinates (x,y) of the given point, where the subscripts l and u denote lower and upper boundary, respectively shall be explained in the particular case later. Let us note that we have chosen the lower surface to have $z=0$, so we do not write it. Accordingly, we can write down the following relation

$$\left(\frac{dE}{dt}\right)_l = \iint_S dxdy \int_{\varphi_l(\vec{r})}^{\varphi_u(\vec{r})} d\varphi \int_{\vartheta_l(\vec{r},\varphi)}^{\vartheta_u(\vec{r},\varphi)} \cos\vartheta \sin\vartheta \ d\vartheta \qquad (5)$$

Combining Eqs. (4) and (5) we can evaluate the loss coefficient k we need for calculating the average albedo given by Eq. (3).

We demonstrate this procedure by applying it to a particular model system consisting of the square grid-cell with the edge size L as presented in Fig. 2. For the simplicity we assume that this grid-cell is divided into two subregions having rectangular form, of areas $S_1 = L \times l$ and $S_2 = L \times (L - l)$, with corresponding albedos α_1 and α_2, respectively, while relative height of higher surface is h. Now $(dE/dt)_h = I \, L \, l \, \pi$ while $(dE/dt)_l$ given by Eq. (5) becomes

$$\left(\frac{dE}{dt}\right)_l = I \int_0^l dy \int_0^L dx \int_{\varphi_{lg}}^{\varphi_u} \int_{\vartheta_l}^{\vartheta_u} \cos\vartheta \sin\vartheta \ d\vartheta \ d\varphi \qquad (6)$$

with

$$\varphi_l = arctg \frac{l-y}{L-x} \qquad \varphi_u = \frac{\pi}{2} + arctg \frac{x}{l-y} \qquad (7)$$

and

$$\vartheta_l = arctg \frac{l-y}{h \sin\varphi} \qquad \vartheta_u = \frac{\pi}{2} \qquad (8)$$

as defined in Figs. 2 and 3.

Let us note that the expression for ϑ_l is valid for any $0 \le \varphi \le \pi$.

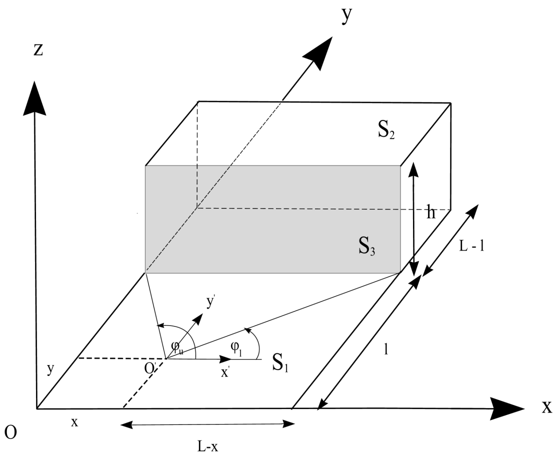

Figure 2. Schematic representation of the square grid-cell consisting of two surfaces of the relative height h and definition of the boundaries for the integration over the local azimuthal angle for the square grid-cell. Notation follows the text.

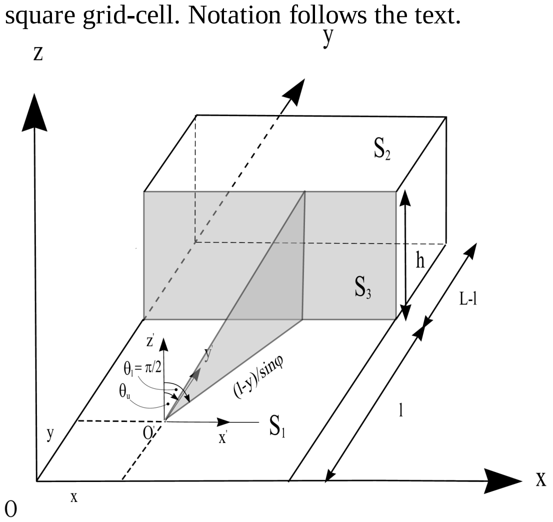

Figure 3. Definition of the boundaries for the integration over the local zenithal angle for the square grid-cell.

Introducing the reduced dimensionless quantities

$$\hat{x} = \frac{x}{L}, \quad \hat{y} = \frac{y}{L}, \quad \hat{l} = \frac{l}{L}, \quad \hat{h} = \frac{h}{L} \tag{9}$$

our final result for the loss coefficient (4), as the function of the reduced relative height $\hat{h}$ and reduced length $\hat{l}$, after some transformations can be presented as

$$\begin{aligned} k(\hat{l},\hat{h}) = \frac{1}{\hat{l}\,\pi}\Big\{ & \hat{l}\; arctg\frac{1}{\hat{l}} - \sqrt{\hat{h}^2+\hat{l}^2} - \\ & - \sqrt{\hat{h}^2+\hat{l}^2}\; arctg\frac{1}{\sqrt{\hat{h}^2+\hat{l}^2}} + \hat{h}\; arctg\frac{1}{\hat{h}} + \\ & + \frac{1}{4}(1-\hat{l}^2)\left[\ln(1+\hat{l}^2) - \ln(1+\hat{h}^2+\hat{l}^2)\right] + \\ & + \frac{1}{4}\hat{h}^2 \ln(1+\hat{h}^2+\hat{l}^2) + \\ & + \frac{1}{4}(1-\hat{h}^2)\ln(1+\hat{h}^2) + \\ & + \frac{1}{4}\hat{l}^2\left[\ln\hat{l}^2 - \ln(\hat{h}^2+\hat{l}^2)\right] + \\ & + \frac{1}{4}\hat{h}^2\left[\ln\hat{h}^2 - \ln(\hat{h}^2+\hat{l}^2)\right]\Big\}. \end{aligned} \tag{9}$$

This ends our derivation.

3. CONCLUSIONS

two approaches given by expressions (2) and (3) analyzing some limiting cases. The expression (9) vanishes identically for $h = 0$. For $\hat{l} \to 0$, it has a finite value equal $1/2$, and since σ_1 vanishes, average albedo tends to α_2, as it should. For further analysis we have calculated the ratio of two average albedos defined as

$$\Gamma = \frac{\bar{\alpha}_n}{\bar{\alpha}_c} \quad . \tag{10}$$

In the particular case $\alpha_1 = \alpha_2 /2$ this ratio becomes

$$\Gamma = \frac{1-\left[1+k\left(\hat{l},\hat{h}\right)\right]\frac{\hat{l}}{2}}{1-\frac{\hat{l}}{2}} \tag{11}$$

where $k(\hat{l},\hat{h})$ is given by expression (9). We have chosen to consider the reduced relative height $\hat{h} = h/L$ as the parameter and plot Γ as a function of the reduced length $\hat{l} = l/L$ (Fig. 8). The inspection of this figure indicates that the albedo calculated on the proposed way is always lower than on the conventional one, decreasing non-linearly when $\hat{l}$ increases. Also, it is seen that albedo rapidly goes down since Γ decreases up to *20* percent for $\hat{l}$ *=1*. It seems that the proposed method in calculating the albedo may have remarkable consequences on numerical treatment of energy budget over the grid-cell what will be demonstrated in the next section.

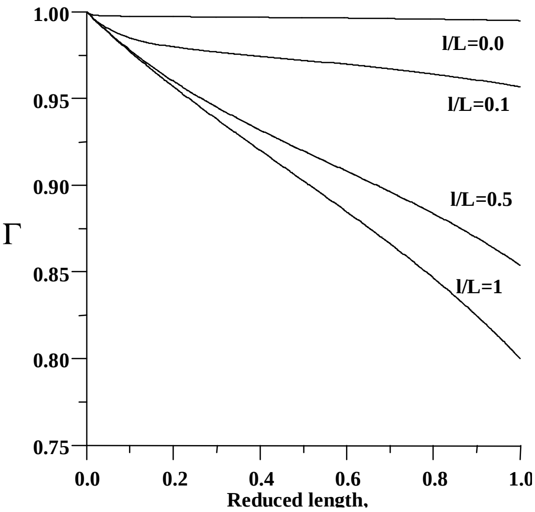

Figure 4. Dependence of Γ ratio on the reduced length *l/L*.

Figure 4 depicts another important property i.e. that for *l = L*, the loss coefficient does not vanish, but in fact remains finite with a value coming from

$$\lim_{\hat{l}\to 1} k(\hat{l},\hat{h}) = \frac{1}{4} + \frac{1}{\pi}\left(\sqrt{1+\hat{h}^2}\, arctg\sqrt{1+\hat{h}^2} - \hat{h} arctg\hat{h}\right) - \frac{1}{2}\left(\sqrt{1+\hat{h}^2} - \hat{h}\right) - \frac{1}{4\pi}\hat{h}^2 \ln\frac{(1+\hat{h}^2)^2}{(2+\hat{h}^2)\hat{h}^2}. \quad (12)$$

This is the consequence of the fact that a vertical surface at the edge of the grid-cell must have an impact to its albedo. We can demonstrate this in a more extreme case by considering the albedo of a square grid-cell surrounded by vertical boundaries of height *h* . If we neglect the boundaries, its albedo would be equal to some value α. However, it can be seen very easily that due to the additivity of solid angles, the effective albedo is equal to α multiplied by the factor $\Lambda = 1 - 4k(1,\hat{h})$, i.e. a factor whose magnitude is between *0* and *1*. In fact, for *h = 0*, it is equal to *1*, while for a large *h* it vanishes. This factor expression is plotted in Fig. 9, in terms of the reduced relative height $\hat{l} = l/L$. On should notice the importance of this effect for the calculations in urban areas, where the height might be close or even larger than the cell size, so *h* need not be small at all.

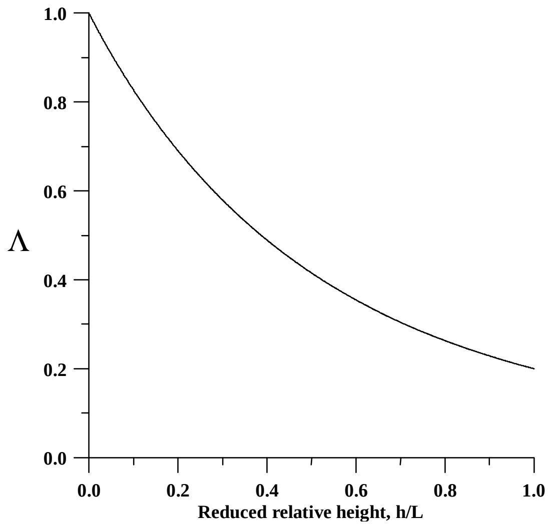

Figure 5. Dependence of Λ factor expression on the reduced relative height *h/L*.

For the small values of the relative height *h* the curve in Fig. 5 has a linear slope. Consequently, the loss coefficient is proportional to h what allows us in practical calculations in environmental modeling to use rather a simplified form of loss coefficient instead of its complete form given by the expression (5). In fact dimensional considerations indicate that this must be true in the most general case, so that in our calculations in progress, we use this approximation to study some practical situations.

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