iEMSs 2008: International Congress on Environmental Modelling and Software
Integrating Sciences and Information Technology for Environmental Assessment and Decision Making
4th Biennial Meeting of iEMSs, http://www.iemss.org/iemss2008/index.php?n=Main.Proceedings
M. Sànchez-Marrè, J. Béjar, J. Comas, A. Rizzoli and G. Guariso (Eds.)
International Environmental Modelling and Software Society (iEMSs), 2008

Two Interacting Environmental Interfaces: Bifurcation in Coupled Asymmetric Logistic Maps

Dragutin T. Mihailovic
Faculty of Agriculture, University of Novi Sad, Dositej Obradovic Sq. 8,
21000 Novi Sad, Serbia
(guto@uns.ns.ac.yu)

Abstract: The field of environmental sciences is abundant with various interfaces and it is the right place for application of new fundamental approaches leading towards better understanding of environmental phenomena. For example, following definition of environmental interface by Mihailovic and Balaž [2007], such interface can be placed between natural or artificially built surfaces and atmosphere. In this case, visible radiation provides almost all of the energy received on the environmental interface. Since all the energy transfer processes occurs in the finite time interval the energy balance equation for this environmental interface can be written in terms of finite differences of ground and air temperatures and then, under some conditions, further transformed into the logistic equation [Mihailovic et al., 2001]. In the case of the two interacting environmental interfaces, we have a higher-dimensional complex system. The chaos in a higher dimensional system is one of the focal subject of physics today. One way, in studying this complex system, naturally leads to the model of coupled logistic maps with different strength parameters. In this paper, we report the results of numerical investigation on the system of two logistic maps, representing energy exchange on environmental interfaces, with different strength parameters such that the one map lies in a period one stable attractor or a bifurcation point and the other in chaotic region when decoupled.

Keywords: Chaos; Environmental Interface; Logistic Equation; Energy Balance Equation; Coupled Asymmetric Logistic Maps.

1. INTRODUCTION

The field of environmental sciences is abundant with various interfaces and it is the right place for application of new fundamental approaches leading towards better understanding of environmental phenomena. We defined the environmental interface as an interface between the two either abiotic or biotic environments which are in a relative motion exchanging energy through biophysical and chemical processes and fluctuating temporally and spatially regardless of its space and time scale [Mihailovic and Balaž, 2007]. This definition broadly covers the unavoidable multidisciplinary approach in environmental sciences and also includes the traditional approaches in sciences that are dealing with the environmental space. The environmental interface as a complex system is a suitable area for occurrence of the irregularities in temporal variation of some physical or biological quantities describing their interaction [Rosen, 1991; Selvam and Fadnavis, 1998; Zupanski and Navon, 2007]. For example, such interface can be placed between: human or animal bodies and surrounding air, aquatic species and water and air around them, natural or artificially built surfaces (vegetation, ice, snow, barren soil, water, urban communities) and atmosphere. The environmental interface of different media was recently considered for different purposes [Glazier and Graner, 1993; Nicolov et al., 1995; Martins et al., 2000; Niyogi and Raman, 2001]. In these systems visible radiation provides almost all of the energy received on the environmental interface. Some of the radiant energy is reflected back to space. The interface also radiates some of the energy received from the sun. The quantity of the radiant energy remaining on the environmental interface is the net radiation, which drives certain physical processes important to us. Since all the energy transfer processes occurs in the finite time interval the energy balance equation at any environmental interface can be written in terms of finite differences of ground and air temperatures and then, under some condition, further transformed into the logistic equation [Mihailovic et al., 2001].

The chaos in higher dimensional system is one of the focal subjects of physics today. Along with the approach starting from modeling physical system with many degrees of freedom, there emerged a new approach, developed by Kaneko [1983] to couple many one-dimensional maps to study the behavior of the system as a whole. However, this model can only be applied to study the dynamics of a single medium such as the pattern formation in a fluid. What happens if two media border on each other like environmental interface? One may naturally leads to the model of coupled logistic maps with different strength parameters. Thus it is appropriate to inquire whether loosening the condition of strict identity might bring any new feature while keeping both maps to be logistic to hold the redundant controlling parameters minimal. Even two logistic maps coupled to each other may serve as the dynamical model of driven coupled oscillators [Midorikawa et al., 1995]. It has been found that two coupled identical maps possess several characteristic features which are typical of higher dimensional chaos. In this paper, we report the results of numerical investigation on the system of two logistic maps, representing energy exchange on environmental interfaces, with different strength parameters.

2. ENERGY BALANCE EQUATION FOR ENVIRONMENTAL INTERFACE

We see the outside world, i.e. the world of phenomena (*ambience*) from the observer's perspective (its inner world). In the ambience, there are *systems* of different levels of complexity and their environments. *System* in the ambience is a collection of precepts while whatever lies outside, like the component of a set, constitutes the environment [Rosen, 1991; Sivertsen, 2005]. The fate of science lies in the fact that it is focused on the system [Rosen, 1991]. Furthermore, to anticipate anything, the system is described by states (determined by observations) while the environment is characterised through its effects on the system.

Our basic equation is the energy balance equation. Since all the energy transfer processes occur in the finite time interval we shall immediately write this equation in terms of finite differences i.e. in the form of difference equation

$$\mathbf{D}T_i = \mathbf{F}_n \qquad (1)$$

where $\mathbf{D}$ is the finite difference operator defined as $\mathbf{D}T_i = (T_{i,n+1} - T_{i,n} / \mathbf{D}t$, T_i is the environmental interface temperature, n is the time level, $\mathbf{D}t$ is the time step, $\mathbf{F}_n = (R_n - H_n - E_n - S_n)/c_i$ is defined at the *n*th time level, R is the net radiation, H , and E are the sensible and the latent heat, respectively, transferred by convection, and S the heat transferred by conduction into deeper layers of underlying matter while c_i is the environmental interface soil heat capacity per unit area,. Eq. (1) is also written in the finite difference form from an additional reason. It can be explain if we follow comprehensive consideration by van der Vaart [1973] about replacing given differential equations by appropriate difference equations in modelling of phenomena in physical and biological world. According to its many mathematical models for physical and biological problems have been and will be built in the form of differential equations or systems of such equations. With the advent of digital computers one has been able to find (approximate) solutions for equations that used to be intractable. Many of the mathematical techniques used in this area amount to replacing the given differential equations by appropriate difference equations, so that extensive research has been done into how to choose appropriate difference equations whose solutions are "good" approximations to the solutions of the given differential equations.

In Eq. (1) the sensible heat is calculated as $C_H(T_i - T_a)$ where C_H is the sensible heat transfer coefficient and $T_a(t)$ is the gas temperature given as the upper boundary condition. The heat transferred into underlying soil matter is calculated as $C_D(T_i - T_d)$ where C_D is the heat conduction coefficient while $T_d(t)$ is the temperature of deeper layer of underlying matter that is given as the lower boundary condition. Following Bhumralkar [1975] and Holtslag and van Ulden [1975] the net radiation term in can be represented as $C_R(T_i - T_a)$ where C_R is the radiation coefficient. According to Mihailovic et al. [2001], for small differences of T_a and T_d, the expression for the latent heat can be written in a form $C_L f(T_a)[b(T_i - T_a) + b^2(T_i - T_a)^2]/2$. Here C_L is the latent heat transfer coefficient, $f(T_a)$ is the gas vapor pressure at saturation and b is a constant characteristic for a particular gas. Calculation of time dependent coefficients C_R, C_H, C_L and C_D can be found in Monteith and Unsworth. [1990]. After collecting all terms in Eq. (1) we get

$$\mathbf{D}T_i = A_1(T_{i,n} - T_{a,n}) - A_2(T_{i,n} - T_{a,n})^2 - A_3(T_{i,n} - T_{d,n}) \tag{2}$$

where $A_1 = [C_R - C_H - C_L bf(T_a)]/c_i$, $A_2 = b^2 f(T_a)/(2c_i)$ and $A_3 = C_D / c_i$ are coefficients also depending on $\mathbf{D}t$ .With $\mathbf{D}t_p = 1/(A_1 - C_D / c_i)$ we indicate the scaling time range of energy exchange at the environmental interface including coefficients, that express all kind of energy reaching and departing the environmental interface. For any chosen time interval, for solving Eq. (2), there always exists $\mathbf{D}t_{p,l} = \min[\mathbf{D}t_p(c_i, C_R, C_H, C_L)]$ when energy at the environmental interface is exchanged in the fastest way by radiation, convection and conduction. If we define dimensionless time $\tau = \mathbf{D}t / \mathbf{D}t_{p,l}$ and use lower boundary condition, $T_{d,n} = T_{a,n} - (c_i / C_D)\mathbf{D}T_a$, then Eq. (2) after some transformations takes the form of the logistic equation, i.e.

$$\Gamma_{a+1} = \beta\Gamma_n(1 - \Gamma_n) \tag{3}$$

where the symbols introduced have the following meaning: Γ is the dimensionless quantity [Mihailovic et al., 2001] while $\beta = 1 + \tau$ taking values in the interval $1 < \beta < 4$.

3. TWO INTERACTING ENVIRONMENTAL INTERFACES

Under the aforementioned conditions Eq. (3) represents energy exchange at the uniform environmental interface. However, in the nature usually we encounter mixture of two environmental interfaces, for example, like surface covered by the sparse vegetation. In this case there are two different environmental interfaces. One is barren soil while the second one is the vegetative part. In this case of two interacting surfaces the energy exchange is more complex because it has to be described with two equations having the form of Eq. (3).

The system we study is two linearly coupled maps

$$x_{n+1} = (1 - \varepsilon) f(\mu, x_n) + \varepsilon f(\nu, y_n) \tag{4}$$

$$y_{n+1} = \varepsilon f(\nu, x_n) + (1 - \varepsilon) f(\nu, y_n) \tag{5}$$

where the map $f(\mu, x) = \mu x(1 - x)$ is taken to be the logistic map with strength parameters μ while ε is the coupling parameter. In case of $\mu = \nu$, two maps soon become synchronized no matter what initial conditions may be, i.e., coupled maps are identical with a single logistic map. The interesting is the case of $\mu \neq \nu$. In the following we fix the strength parameters above and below critical value $\mu_* = 3.56994$ for of μ, and ν respectively. We choose the strength parameters μ and ν and regard the coupling parameter ε as the controlling parameter. In Figs. 1 and 2, the attractors of the coupled-map are displayed as functions of coupling ε. Fig. 1 shows the result of $\mu = 4$ and $\nu = 3$, and Fig. 2 that of $\mu = 4$ and $\nu = 2$. In both cases for each value of ε, we used the final value of the previous ε and 1500 iterations are plotted. They are two typical examples of various values of μ and ν. One immediately notices several interesting features. The fact that there are two chaotic regions in both ε = 0 and ε = 1 ends seems odd at first sight, but after some reflection, one realizes that very weak ε means very strong ($1-\varepsilon$) which brings chaos first to the variable x, then to y however weak the coupling term may be. The most salient feature is the appearance of stable period four cycle right after the period one around ε = 0.77 in Fig. 1. Another, found in both (Fig. 1) and (Fig. 2) cases, is the sudden filling of the x and y space around ε = 0.85 and above. The broad window-like region with period four around ε = 0.9 in the case of (Fig. 2) is also noteworthy.

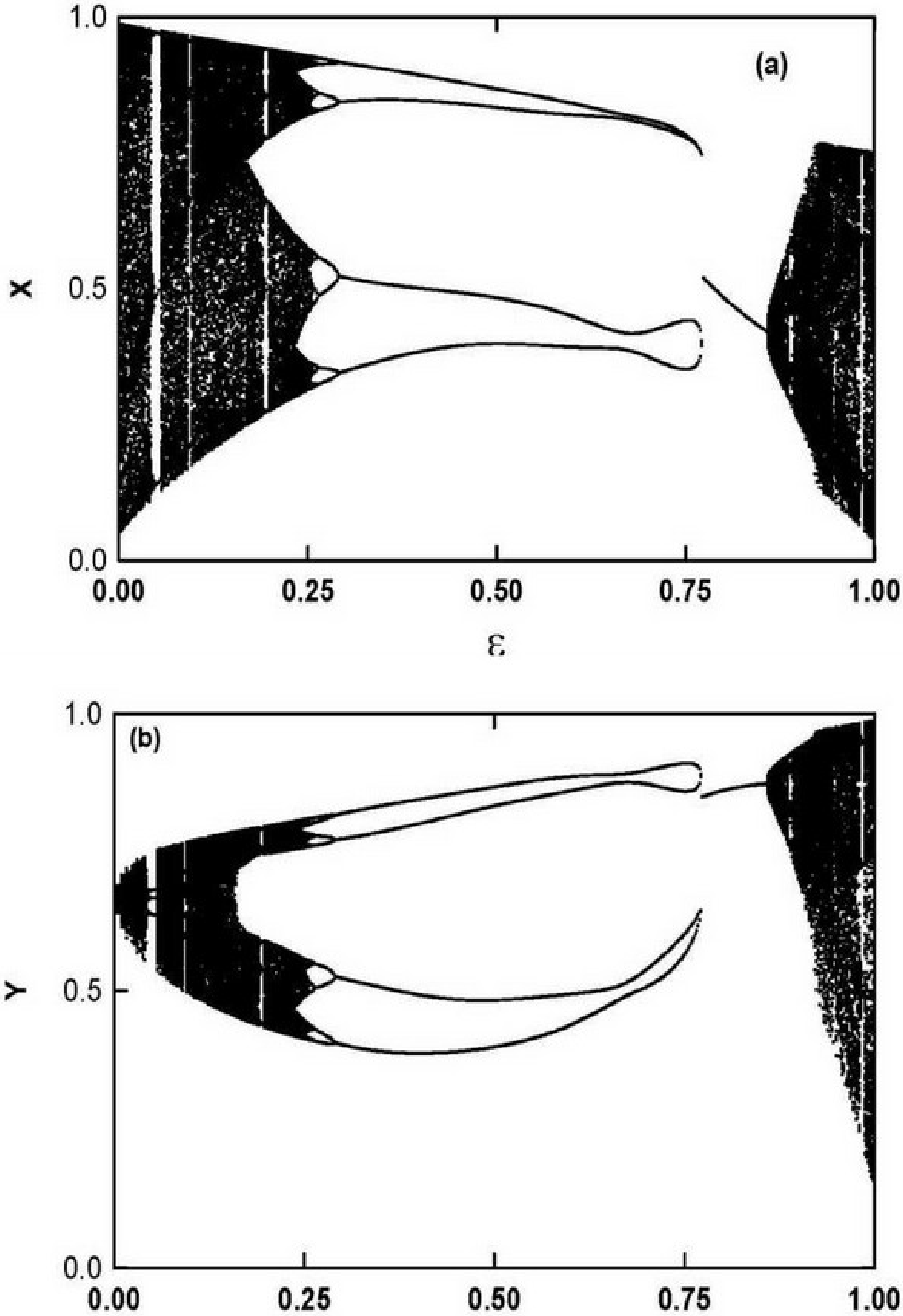

Figure 1. The phase diagram of the coupled maps Eqs. (4)-(5) for parameter values $\mu = 4$, $\nu = 3$; (a) is for x and (b) is for y.

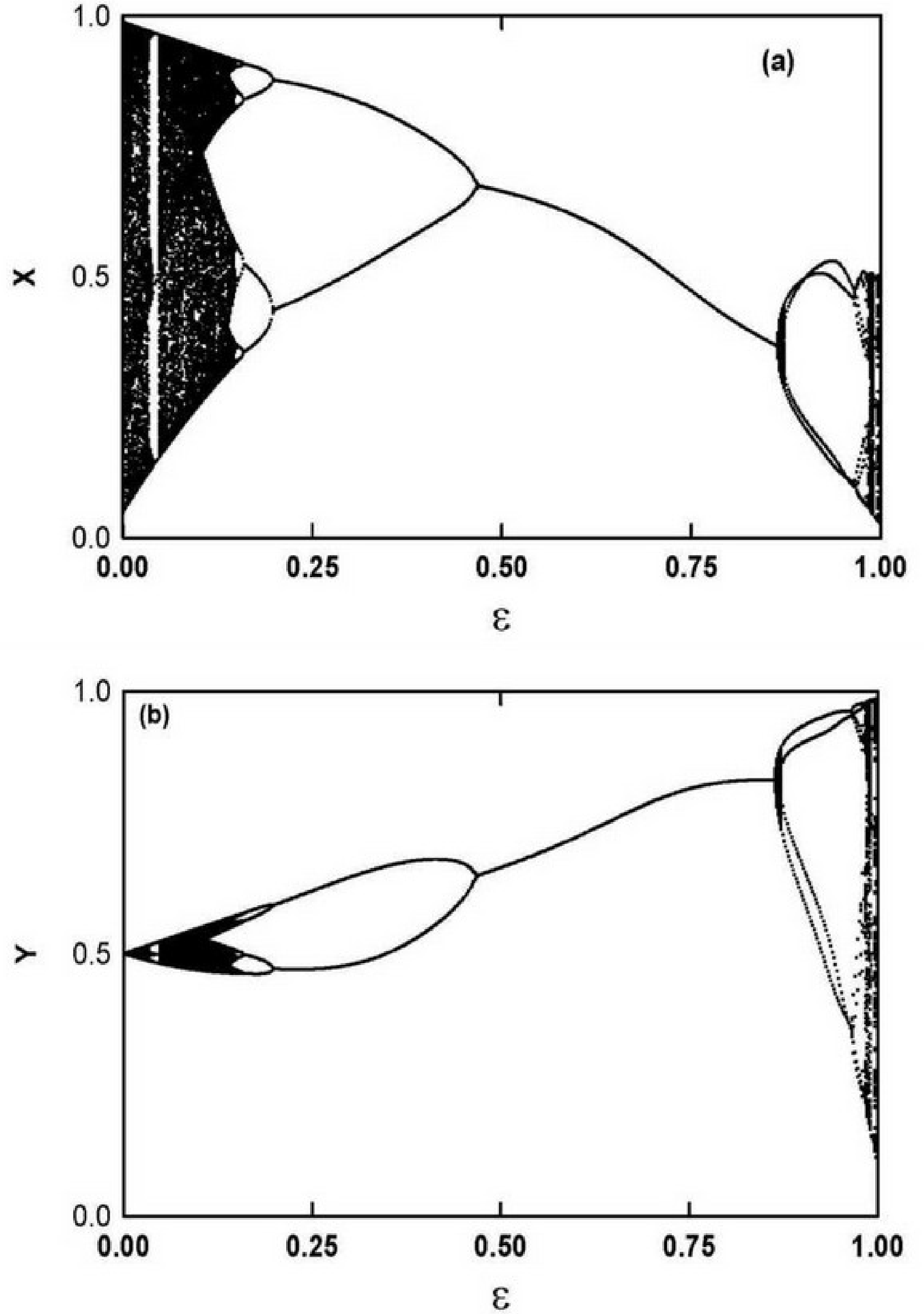

Figure 2. The phase diagram of the coupled maps Eqs. (4)-(5) for parameter values $\mu = 4$, $\nu = 2$; (a) is for x and (b) is for y.

The asymptotic behaviour of a series of iterates of the map can be characterised by the largest Lyapunov exponent which, any initial point $\vec{x}_0$ in an attracting region, is defined by

$$\lambda = \lim_{N\to\infty} \left\{ \ln[\left\| \underline{\mathbf{D}}^{(N)}(\vec{x}_0) / N \right\|] \right\} \qquad (6)$$

where $\left\|\underline{\mathbf{D}}^{(N)}\right\|$ is the norm of the derivative matrix, $\vec{x}_n$ comes from general vector mapping while N is the number of the last iterate. We calculated the Lyapunov exponent λ to see the behaviour of the coupled maps given by Eqs. (4)-(5) depending on different values of coupling parameter ε. Figure 3 shows Lyapunov exponent for the coupled maps as a function of ε ranging from 0 to 1. Each point was obtained by iterating many times from the initial condition to eliminate transient behaviour and then averaging over another 50 000 iterations starting from initial condition $x = 0.2, y = 0.25$ with 500ε values.

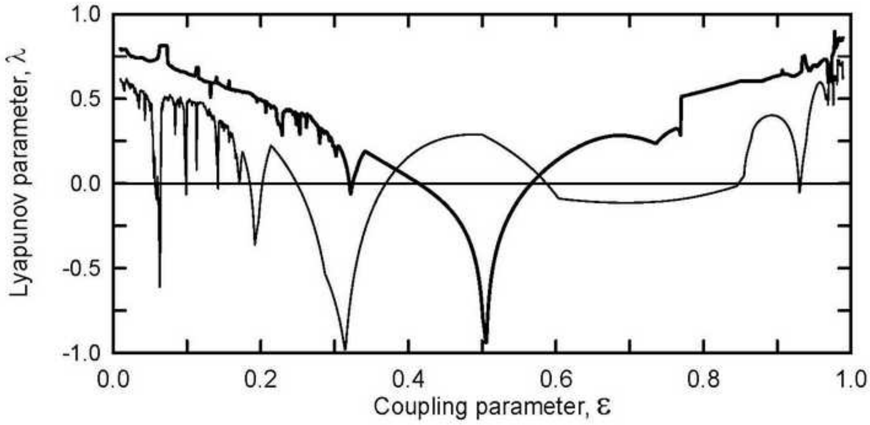

Figure 3. Lyapunov exponent of the coupled maps as a function of coupling parameter ε ranging from 0 to 1 for parameter values $\mu = 4$, $\nu = 3$ (thick line) and $\mu = 4$, $\nu = 2$ (thin line).

4. CONCLUSION

We reported the results of numerical investigation on the system of two coupled logistic maps, representing energy exchange of two interacting environmental interfaces. It has been done by calculating the phase diagrams of the coupled maps for different values of the strength parameters as well as calculation of the Lyapunov exponent as a function of the coupling parameter. It seems that further analysis of this system will be useful for understanding the process of energy exchange between two interacting environmental surfaces.

ACKNOWLEDGEMENT

The research work described in this paper has been funded by the Serbian Ministry of Science and Technology under the project "Modelling and numerical simulations of complex physical systems ", No. ON141035 for 2006-2010.

REFERENCES

Bhumralkar, C.M.,Numerical experiments on the computation of ground surface temperature in an atmospheric general circulation model. *J. Appl. Meteorol. 14*, 1246-1258, 1975.

Glazier, J., and Graner, F., Simulation of the differential adhesion driven rearrangement of biological cells, *Phys. Rev. E*, 47, 2128 – 2154, 1993.

Holtslag, A.A., and A.P. van Ulden, 1975: A simple scheme for daytime estimates of the surface fluxes from routine weather data. *J. Appl. Meteor.*, 22, 517-529.

Kaneko, K., Transition from Torus to Chaos Accompanied by Frequency Lockings with Symmetry Breaking— In Connection with the Coupled-Logistic Map, *Prog. Theor. Phys.* 69(5), 1427-1442, 1983.

Martins, M.L., Ceotto, G., Alves, G., Bufon, C.C.B, Silva, J.M., and Laranjeira, F.F., Cellular automata model for citrus variegated chlorosis, *Phys. Rev. E*, 62, 7024 – 7030, 2000.

Midorikawa, S., Takayuki, K., and Taksu C., Folded bifurcation in coupled asymmetric logistic maps*, Prog. Theor. Phys.*, 94, 571-575, 1995.

Mihailovic, D.T, and Balaž, I., An essay about modelling problems of complex systems in environmental fluid mechanics, *Idojaras*, 111(2-3), 209-220, 2007.

Mihailovic, D.T., Kapor, D.V., Lalic, B., and Arsenic, I., The chaotic time fluctuations of ground surface temperature resulting from energy balance equation for the soil-surface system, *Abstracts of the 26th General Assembly of European Geophysical Society*, March 20-25. Nice, France, 2001.

Monteith, J.L., Unsworth, M., Principles of Environmental Physics, Second Edition, Elsevir, p. 304, 1990.

Nikolov, N., Massman, W., Schoettle, A., Coupling biochemical and biophysical processes at the leaf level: an equilibrium photosynthesis model for leaves of C3 plants, *Ecol. Modelling*, 80, 205-235, 1995.

Niyogi, D.S., and Raman, S., *Numerical modeling of gas deposition and bidirectional surface–atmosphere exchanges in mesoscale air pollution systems*. In: Mesoscale Atmospheric Dispersion, Ed. Z. Boybeyi, WIT Publications, Southampton, UK, Advances in Air Pollution, Vol. 9, 1-51, 2001.

Rosen, R., *Life itself: A comprehensive inquiry into the nature, origin, and fabrication of life.* Columbia University Press. New York, pp. 285, 1991.

Selvam, A.M., and Fadnavis, S., Signatures of a universal spectrum for atmospheric interannual variability in some disparate climatic regimes, *Meteor. Atmos. Phys.*, 66(1-2), 87-112, 1998.

Sivertsen, T.H. Discussing the scientific method and a documentation system of meteorological and biological parameters. *Phys. Chem. Earth.* 30:35-43, 2005.

van der Vaart, H.R., A comparative investigation of certain difference equations and related differaential equations: Implications for model building. *Bull. of Math. Biol.,* 35, 195-211, 1973.

Zupanski, M., and Navon, I.M. Predictability, observations, and uncertainties in the geosciences. *Bull. Amer. Meteor. Soc.*, 88, 1431–1433, 2007.