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Action Prioritization to Address the Silao-Romita Aquifer Problem through the Analytic Hierarchy Method

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Abstract: The Silao-Romita aquifer is located on the west-central state of Guanajuato, Mexico. The main problems of this area are over-exploitation and pollution, which cause other specific problems. The objective of the present work is to analyze the current situation to establish a new prioritization of action plans. To this purpose, we use the analytic hierarchy process (AHP) with the participation of one of the main actors involved. In AHP the input format for decision-makers to express their preferences regarding criteria and alternatives derives from pair-wise comparisons to build the comparison matrices. The matrix of criteria shows how much importance a criterion is given compared with another criterion with respect to the interests or preferences of the respondents. Achieving a good level of consistency is crucial, since only priorities derived from consistent judgment are reliable.

***Keywords**:* analytic hierarchy process, action prioritization, aquifer over-exploitation and pollution.

1 INTRODUCTION

This paper presents an alternative to mainstream experience in decision-making processes in engineering projects. To this purpose we use the method of analytic hierarchies (AHP) [Saaty, 1977; 1980] in a problem regarding the Silao-Romita aquifer, which suffers from various problems, mainly derived from over-exploitation and pollution.

One of the challenges to address in this general situation is the different point of view of the actors involved. There is a need for action aimed for solving the general situation of the aquifer. However, since each one of those sectors involved has a different perception, an appropriate mechanism to promote a consensus on perception of the problem is required. For encouraging the generation of possible solutions to efficiently manage the available resources, the State Water Commission (CEAG in Mexican) promoted the creation of groundwater users groups. One member of the Silao-Romita group acted as an expert in this study.

In AHP experts' judgment is compiled into so-called comparison matrices, and, then, calculation of the priorities of the elements is performed. There are many applications of AHP in different fields, the most common being in the areas of logistics, manufacturing, government and education. In this paper a partial implementation of the AHP method to analyze the problem of Silao-Romita aquifer, located in central Mexico, is described. Partial application is used in the following sense. In general, the complete AHP involves not only comparisons among the criteria, but also comparisons of various alternatives with regard to each criterion, ending with a final process of aggregation of priorities. In the case we present here, the main objective was to prioritize a number of possible actions to be taken to solve the problem of the aquifer. Interestingly, just the prioritization of these alternatives emerged as a conclusive elucidation regarding the lines of action to be addressed. This fact, together with a process to streamline the trade-off between expert(s) know-how and synthetic consistency described in the next section, are the main novelties of this application.

2 ANALYTIC HIERARCHY PROCESS

The analytic hierarchy process (AHP) was developed by Thomas L. Saaty [1977] and is intended to formalize the intuitive understanding of complex problems by building a hierarchical model. The purpose of the method is to allow the decision maker to structure a multicriteria problem visually, by building a hierarchical model that basically has three levels: goal or objective, criteria and alternatives. Once constructed the hierarchical model, pairwise comparisons between these elements (criteria and alternatives) are complied into square matrices, whose coefficients are numerical values assigned to the preferences indicated by experts. The process ends by providing a synthesis of the same by aggregation of such opinions.

The foundation of the process is based on the fact that it allows to give numerical values to the judgments given by the people, which helps measure how much each element of the hierarchy affects the next level. For these comparisons use is sometimes made of the scale developed by Saaty [1980; 2001], contained in Table 1. However, there are some studies on the development of other scales of values [Dong *et al.,* 2008]. It should be noted that the scale can be extended to the use of intermediate values, considering that a judgment is between two possible values in the table. Comparisons between pairs are quantified by the scale available to the decision maker. The scale lists a number of verbal opinions and a discrete set of numbers which represent the importance or weight of verbal opinions.

This methodology requires to handle problems where the number of comparative elements is at most $n = 7 \pm 2$ [Miller, 1955]. In the case of exceeding that number, it is suggested the composition of clusters, allowing the creation of a set of elements using eventually relative measures [Escobar and Moreno, 1997].

Table 1. Scale comparison elements.

Verbal judgment	Scale ($a_{i,j}$)
Absolute importance of element *i* over element *j*	9
Very strong importance of the element *i* over element *j*	7
Marked importance of element *i* over element *j*	5
Small importance of element *i* over element *j*	3
Equal importance or indifference between *i* and *j*	1
Small importance of element *j* over element *i*	1/3
Marked importance of element j on element i	1/5
Very strong importance of the element *j* over element *i*	1/7
Absolute importance of element *j* over element *i*	1/9

In the first step the expert makes a comparison between pairs of elements. When performing pairwise comparisons between the elements involved a square matrix, $A_{n\times n}$, is built, where matrix entry (i,j) is a number representing the comparison between elements i and j, according to the scale used; n is the number of elements compared.

All the considerations for the construction of comparison matrices apply equally to the case of the comparison matrix of criteria and for comparing the matrices of alternatives for each criterion.

2.1 Some preliminary questions about consistency

Let us first recall the basic facts about comparison matrices.
We give first some definitions. Consider a real matrix A of size $n \times n$. A is positive if $a_{ij} > 0$ for all i, j; A is homogeneous if $a_{ii} = 1$ for all i; A is reciprocal if $a_{ij} = 1/a_{ji}$, for all i, j. These are typical properties of comparison matrices typically found in AHP. In addition, A is consistent if $a_{ik} = a_{ij}a_{jk}$, for all i, j, k.
Among the various characterizations of consistent matrices, we recall the one given in [Benítez *et al*. 2011a]: a positive matrix A is consistent if and only if there exists a vector x in R^n such that $A = xJ(x)^T$, where J is the map associating a positive matrix $X = (x_{ij})$ with the matrix whose entry (i, j) is $1/x_{ij}$. (Recall that if X is any matrix, X^T denotes the transpose of X). This characterization is used to build the consistent matrix that is closer to a given comparison matrix, once a suitable prioritization vector has been obtained. This prioritization vector is closely related with the so-called Perron vector of a positive matrix.
The principal eigenvalue of a comparison matrix and its associated eigenvector (Perron vector) provide information for complex decision making: the normalized Perron eigenvector provides the priority vector sought [Saaty 2003, 2008]. In the general case, however, A is not consistent. The hypothesis that the estimates of these values are small perturbations of the 'correct' values also guarantees a small perturbation of the eigenvalues (see for example [Stewart, 2001]). For non consistent matrices, the problem to be solved is the eigenvalue problem $Aw = \lambda_{max}w$, where λ_{max} is the single largest eigenvalue of A, which provides the Perron eigenvector as an estimate of the vector of priorities. As a measure of the inconsistency, Saaty proposes to use the consistency index $CI = (\lambda_{max} - n)/(n - 1)$ and the consistency ratio $CR = CI / RI$, where RI is the Saaty's average consistency index [Saaty, 2001]. If $CR < 0.1$, the estimate is accepted, otherwise, a new comparison matrix is requested until $CR < 0.1$.

2.2 Linearization process

Several alternatives have been proposed in the literature to help improve consistency. In this case we use a technique based on a linearization technique [Benítez *et al*. 2011a] together with an iterative feedback process to achieve an acceptable level of consistency while complying to some degree with expert preferences.
The process starts with a comparison matrix provided by the expert(s). Most comparison matrices typically are non-consistent. Even more, with non-negligible probability most comparison matrices do not have acceptable consistency ratios. Then, various prioritising processes, in particular, the proposed linearization technique, can be used to build a suitable consistent matrix. However, with non-negligible probability, the new matrix thus generated may be considered by the expert(s) to only partially reflect their opinions, and they may choose to modify some of the matrix entries. Shifting one or more entries of the matrix – while preserving reciprocity – will produce a new inconsistent matrix and a similar process can again be undergone in an attempt to reach a reasonable trade-off between consistency and expert know-how compliance.
We just give here a concise enunciation of the linearization process. This process [Benítez *et al*. 2011a] states that the closest matrix to an $n \times n$ consistent comparison matrix A can be obtained through the orthogonal projection of $L(A)$ onto $\mathcal{L}_n = \{L(A) : A \; n \times n \text{ positive consistent matrix}\}$, a subspace of dimension $n - 1$ of the space of $n \times n$ matrices. L associates a positive matrix $X = (x_{ij})$ with the matrix whose (i,j) element is $\log(x_{ij})$. This orthogonal projection is given by a suitable Fourier expansion

$$p_n(L(A)) = \frac{1}{2n} \sum_{i=1}^{n-1} \frac{trace(L(A)^T \phi_n(y_i))}{\|y_i\|_2^2} \phi_n(y_i) \, ,$$

where ϕ_n is defined by $\phi_n(v) = v1_n^T - 1_n v^T, v \in R^n$, with 1_n the vector $(1 \ldots 1)^T$ in R^n, and $\{y_1, \ldots, y_{n-1}\}$ is an orthogonal basis of the orthogonal complement of the linear span of 1_n. Finally, the closest consistent matrix to A is given by $E(p_n(L(A)))$, where E associates a matrix $X = (x_{ij})$ with the matrix whose (i,j) entry is $\exp(x_{ij})$.

3 APPLICATION

The Silao-Romita aquifer is located in the central-western state of Guanajuato in central Mexico; it includes the Guanajuato River sub-basin and has an approximate area of 195.242 ha. The general problem of the study area is over-exploitation and pollution of the aquifer, resulting in the emergence of several specific problems. One of the main challenges to address this general problem is the different viewpoints of the sectors involved. There is a need for action aimed at solving the general problem of the aquifer. However, as each involved stakeholder has a different perception, there is a clear need of appropriate mechanisms to achieve consensus in the perception of the problem, which is a clear drawback to solve the main problem.

To influence the generation of alternative solutions and to efficiently manage the available resources, the CEAG promoted the association of users into groups called Groundwater Technical Committees (COTAS in Mexican). Particularly the Silao-Romita COTAS was constituted as a civil partnership in 1999, with the ultimate goal of acting as a promoter and advisor in the issue of efficient use of water. Since its creation a number of steps have been taken aimed at solving the general problem of the aquifer, managing to solve some specific problems. In the present work we analyze the current situation to establish a new prioritization of action plans. To this end, we use the analytical hierarchy methodology with the participation of a board member of Silao-Romita COTAS. This application seeks to lay the groundwork for a global application which includes views of representatives of all users of the aquifer. In this case, we have turned to an expert from COTAS Silao-Romita, which have acted as an advisor from the beginning of this COTAS's creation. This fact supports this expert's knowledge about the current status of the study area, as well as about the tasks and projects that have been conducted during the existence of the committee.

The first step followed for the analysis of the problem, was to resort to the record that existed on the priorities of resource management at the beginning of the work of COTAS. This listing came after several meetings with council members. According to this first list, priorities are listed below, in no particular order:

1. Compile and analyze agreements and laws.
2. Collect and classify user registry.
3. Education on water efficiency: primary schools, users and general public.
4. Repair leaks (urban use).
5. Improve efficiency of irrigation systems.
6. Promote the construction of treatment plants in Silao, Romita and Guanajuato.
7. Reforestation programme.
8. Mechanisms of rainwater harvesting.
9. Construction, rehabilitation and maintenance of levees, canals, river beds and retained water (former dams).
10. Endow the Gavia's dam with irrigation and/or beekeeping activities, and the Chichimequillas' dam with recharging and drinking use.
11. Set extraction limits to each well.
12. Develop a programme of low water use crops.
13. Encourage private participation in irrigation programs.
14. Mechanisms to encourage efficient use and penalties for misuse of water.
15. Define duty and responsibility of government.
16. Monitor city growth of Silao, Romita, Guanajuato and Irapuato.

3.1 Restructuring the problem

After analyzing the results of the first study of the aquifer problems during the last 12 years, the expert was asked to conduct a new list of necessary action plans to address current problems of the users and the area, which is the following:

- **P1.** Collect data and regulations regarding extraction of water at municipal, state and federal levels. - Information center.
- **P2.** Get a user registry. - Database.
- **P3.** Promote technological development of agricultural use. - Modernization.
- **P4.** Make the COTAS can self-finance. - Financing.
- **P5.** Restore and manage natural resources in the area. – Micro-catchments.
- **P6.** Contribute to aquifer regulation. - Regulations.
- **P7.** Communicate results of hydrological studies, work plan and issues of interest to groups, associations, institutions and the general assembly of COTAS. - Dissemination.
- **P8.** Promote the design, construction and maintenance of wastewater treatment plants. - Pollution.
- **P9.** Mediate in water conflicts. - Conflicts.

To assist in carrying out this new list of priorities, regular meetings were held with lecturers and researchers of the Hydraulics and Geomatics Department of the University of Guanajuato, as well as students of the MSc in Hydraulics from the same university. The purpose of considering these people was to have an overview of the problem. An additional advantage of having had the support of these professionals is that the University of Guanajuato is located in the area of Silao-Romita aquifer. Thus, their participation was convenient and rewarding as they have a direct relationship with the object of analysis. Even though the comparison matrix was filled out just by the COTAS' expert, people involved helped clarify the list of nine elements that eventually formed the comparison matrix. These meetings helped achieve a new list of priorities in no specific order.
As a part of the AHP, the expert carried out comparisons between pairs of elements. In this current analysis the matrix in Table 2 was compiled. These comparisons were made just after the list was decided, so this matrix compiles the former view of the expert regarding each matrix element.

Table 2. Original comparison matrix

	P1	P2	P3	P4	P5	P6	P7	P8	P9
P1	1	3	1/3	1/5	1/7	1/9	3	1/7	3
P2	1/3	1	3	1/7	1/7	1/9	3	1/7	3
P3	3	1/3	1	9	3	1/9	3	1/5	5
P4	5	7	1/9	1	9	9	9	9	9
P5	7	7	1/3	1/9	1	1/9	7	7	9
P6	9	9	9	1/9	9	1	9	9	9
P7	1/3	1/3	1/3	1/9	1/7	1/9	1	3	3
P8	7	7	5	1/9	1/7	1/9	1/3	1	5
P9	1/3	1/3	1/5	1/9	1/9	1/9	1/3	1/5	1

Its Perron eigenvector, and the consistency ratio *CR* are as follows:

$$Z = [0.0247\ 0.0450\ 0.1815\ 0.2790\ 0.0901\ 0.2527\ 0.0271\ 0.0906\ 0.0093]^T.$$
$$CR = 75.9\%$$

The highest value is related to the action plan of greater weight and the lowest value is the one with lower weight.
The value of *CR* obtained greatly exceeds the limit allowed for the comparison matrix *A* and the results cannot be considered valid in the decision making process. Nevertheless, according to Aznar and Guijarro [2008], a simple way that can improve the consistency is by reordering the elements according to the weights of the initial comparison matrix: it is easier to compile a comparison matrix

considering elements that have been previously ranked. So these elements were rearranged as follows:

P4. Financing
P6. Regulations
P3. Modernization
P8. Pollution
P5. Micro-catchments
P2. Database
P7. Dissemination
P1. Information centre
P9. Conflicts

Starting from this new rearrangement of elements, the expert was asked to carry out the benchmarking exercise again. The new comparison matrix containing views of the expert in this second iteration is shown in Table 3.

Table 3. Second iteration for the comparison matrix

	P4	P6	P3	P8	P5	P2	P7	P1	P9
P4	1	9	7	5	9	3	3	7	5
P6	1/9	1	5	3	9	5	3	5	5
P3	1/7	1/5	1	9	7	9	3	5	5
P8	1/5	1/3	1/9	1	9	7	5	5	5
P5	1/9	1/9	1/7	1/9	1	5	3	5	5
P2	1/3	1/5	1/9	1/7	1/5	1	5	5	5
P7	1/3	1/3	1/3	1/5	1/3	1/5	1	5	5
P1	1/7	1/5	1/5	1/5	1/5	1/5	1/5	1	5
P9	1/5	1/5	1/5	1/5	1/5	1/5	1/5	1/5	1

For this comparison matrix the Perron eigenvector and the consistency ratio are

$$Z = [0.3576\ 0.1837\ 0.1759\ 0.1084\ 0.0518\ 0.0473\ 0.0385\ 0.0207\ 0.0161]^T,$$
$$CR = 36.5\%.$$

After this iteration, the result of *CR* is significantly reduced, but still fails to fall within the tolerance limit of consistency (10%). It is worth observing that, although the weights have changed, no reversion is observed on the position of each element with respect to the matrix in Table 3.
However, in order for the decision-making process to have the necessary validity, the linearization method developed by Benítez *et al*. [2011a] of improving consistency is applied to find the closest consistent matrix. This method produces the following results:

Table 4. Consistent comparison matrix

	P4	P6	P3	P8	P5	P2	P7	P1	P9
P4	1.00	1.14	1.11	1.12	8.56	5.94	3.70	6.51	6.37
P6	0.88	1.00	0.97	0.98	7.49	5.20	3.24	5.70	5.58
P3	0.90	1.03	1.00	1.01	7.74	5.37	3.34	5.89	5.76
P8	0.89	1.02	0.99	1.00	7.63	5.29	3.30	5.81	5.68
P5	0.12	0.13	0.13	0.13	1.00	0.69	0.43	0.76	0.74
P2	0.17	0.19	0.19	0.19	1.44	1.00	0.62	1.10	1.07
P7	0.27	0.31	0.30	0.30	2.31	1.61	1.00	1.76	1.72
P1	0.15	0.18	0.17	0.17	1.31	0.91	0.57	1.00	0.98
P9	0.16	0.18	0.17	0.18	1.34	0.93	0.58	1.02	1.00

$$Z = [0.2203\ 0.1930\ 0.1993\ 0.1965\ 0.0258\ 0.0371\ 0.0596\ 0.0338\ 0.0346]^T,$$
$$CR = 0\%.$$

The modifications are necessary for the decision making process to be supported by a solid foundation. The values in Table 4 are those with less difference from the values that the expert had expressed in the comparison matrix of Table 3. These values have been analyzed with the expert in order to obtain his approval. The aim is to reach the necessary trade-off between expert judgment and consistency enforcement [Section 2.2]. Then an insightful meeting was held with the expert, where each value on the consistent matrix (table 4) checked. After the interchange of ideas and approaches it was easier to understand value modifications, considering the verbal judgment in table 1. Even though elements' order have been modify after consistency improvement, the expert eventually agreed with the process.

4 RESULTS

According to the expert's opinions, the priority order, as well as the weights of the action plans necessary to solve the problem of the aquifer, are as follows:

Table 5. Results

Orde	Plan		Weight
1	P4	Financing	0.2203
2	P3	Modernization	0.1993
3	P8	Pollution	0.1965
4	P6	Regulations	0.1930
5	P7	Dissemination	0.0596
6	P2	Database	0.0371
7	P9	Conflicts	0.0346
8	P1	Information centre	0.0338
9	P5	Micro-catchments	0.0258

COTAS, as an organization, is concerned primarily on having funds to carry out its mission. The second best plan is technological development of agriculture, because it uses almost 70% of groundwater. Thirdly, pollution management by developing or improving wastewater treatment plants is identified. It means that after this analysis, we confirm that the main problems of Silao-Romita aquifer are exploitation and pollution, as these priorities have not changed. However, after 12 years of COTAS activity, with many undergone changes on water and land use, some priorities have changed, and also new issues have emerged.

5 CONCLUSIONS

Application of AHP technique is helpful to analyze the aquifer problem and establish a priority order of action plans. A brainstorming exercise is necessary to set new problems of the area. A mathematical procedure ensures that ranking process of elements is reliable. It is based on experts' opinions, which are crucial in this decision making process. This partial application of AHP provides the basis for a global exercise that will include the opinions of other COTAS's members, namely, water users, academic and technical experts, among other stakeholders. Furthermore, the present results provide a guideline for implementation of priority actions that help solve the general problem of the study area.

We should mention that the exercise presented in this paper has reinforced the relationship between the University of Guanajuato and the Silao-Romita COTAS. After the work meetings, both associations have started to consider the possibility of implementation of an agreement between them to establish the basis for cooperative research activities on physical and social environments regarding water issues. This agreement will encourage interaction of technical experts of both institutions to promote information exchange, development of projects, and participation of researchers and students in solving the real problems of Silao-Romita aquifer area.

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