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Limitations in Interpretable Top-Down Effective Rainfall-Runoff Modelling

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Abstract: The effective rainfall-flow module of a "top-down" hydrological model has linear, fixed dynamics, giving a two-exponential instantaneous unit hydrograph (IUH) often interpreted as summing slow- and quick-flow components. Such components suffer uncertainty due to ignorance of rainfall variation between samples and ignorance of delay in the IUH. If they are obtained from linear-in-parameters models, ill conditioning in the conversion amplifies the uncertainty. The extent of these uncertainties is analysed, with examples. The paper also considers alternatives to performance assessment of such models by Nash-Sutcliffe efficiency. Rather than employing the sample mean as an output-prediction benchmark, they use almost equally simple predictors taking the flow correlation structure into account. Again numerical examples are given.

Keywords: Hydrology; uncertainty; model calibration; model performance measurement.

1 INTRODUCTION

This paper has two aims. The first is to analyse the possible extent of various errors arising in the parameter estimation of models relating effective (excess) rainfall and river flow (Evans and Jakeman [1998]). Attention is confined to linear, time-invariant (LTI), discrete-time models, calibrated on rainfall and flow records. Section 2 examines error arising from the nature of the modelling process rather than from observational errors or from the model-structure-selection or parameter-estimation technique. The influence of non-linearity in catchment dynamics on uncertainty is not considered, being less amenable to analysis.

The second aim is to seek alternatives to Nash-Sutcliffe efficiency to measure how well a model fits observed flow. Section 3 examines measures based on simple flow predictors.

These topics are not novel; rainfall-flow models are recognised as subject to many sources of error and Nash-Sutcliffe efficiency has been widely criticised ([Croke [2007]; Schaefli and Gupta [2007]). The contributions here are in showing how the ranges of certain errors depend on model characteristics and the observation interval, and in offering less flattering model-performance measures.

2 UNCERTAINTIES IN EFFECTIVE RAINFALL-FLOW MODELS

2.1 Errors in model due to unmodelled features of rainfall and IUH

The next two subsections discuss modelling errors, and hence uncertainties, due to inability of the hydrological model specification to capture features of the underlying physical behaviour. These errors are fundamental and do not depend on details of the model or peculiarities of particular records. Straightforward analysis can establish bounds on the resulting uncertainties in unit-hydrograph components. To establish their distributions or to turn them into uncertainties in a modelled flow time series, distributional properties of the records have to be known, so that is not attempted.

2.2 Error due to ignorance of rainfall distribution over observation interval

Error arises first from the discrete-time nature of the model. The instantaneous unit hydrograph (IUH) is conceptually the continuous-time flow response to unit rainfall arriving all at time zero, i.e. the unit-impulse response $h(t),\ t \geq 0$. The model actually relates rainfall and flow observed synchronously at time intervals T, say. Furthermore, unit input at time zero is rainfall registered as 1 at that instant, i.e. unit total rainfall over $-T < t \leq 0$. The model produces the unit-pulse response (UPR) h_k, $k = 0,1,\ldots,\infty$, of flow values at successive observation instants $t = kT$ resulting from unit rainfall registered at time zero. The response to rainfall registered by an observation depends on how the rainfall varied in the preceding time step. A calibrated model averages such responses over the record. Use of the model incurs errors through this average differing from the response appropriate to each time step of rain. The following analysis examines such errors, for the common interpretation of the UPR of an effective rainfall-flow model (Jakeman and Hornberger [1993]) as representing an IUH with quick- (q) and slow- (s) flow components:

$$h(t) = g_q \exp\left(-t/\tau_q\right) + g_s \exp\left(-t/\tau_s\right),\ t \geq 0 \tag{1}$$

with time constant $\tau_s > \tau_q$. The IUH gives the flow response due to rainfall $u(t)$ in the period $-T < t \leq 0$ as

$$q(t) = \int_{-T+}^{0} u(\tau) h(t-\tau) d\tau,\ t \geq 0 \tag{2}$$

The effects of not knowing the time course of rainfall over an observation interval are readily seen for a single IUH component (component i in general)

$$h_i(t) = g_i \exp\left(-t/\tau_i\right),\ t \geq 0\,. \tag{3}$$

The corresponding actual flow component due to unit rainfall in the observation interval up to time zero is found by substituting this IUH component and the actual rainfall history into (2). Comparing the extremes and an intermediate case, consider rain falling as (i) a very short pulse $u(t) = \delta(-T+)$ at the start of the period, (ii) a very short pulse $u(t) = \delta(0)$ at the end, or (iii) steady rain $u(t) = 1/T$, all for $-T < \tau \leq 0$, where $\delta(t)$ is the Dirac δ function, a unit-area impulse at $t = 0$. The corresponding responses are

$$q_i(t) = \begin{cases} \text{(i)}\ g_i \exp\left(-(t+T)/\tau_i\right) \equiv g_i \exp\left(-T/\tau_i\right)\exp\left(-t/\tau_i\right) \\ \text{(ii)}\ g_i \exp\left(-t/\tau_i\right) \\ \text{(iii)}\ \dfrac{g_i \tau_i}{T}\left(1-\exp\left(-T/\tau_i\right)\right)\exp\left(-t/\tau_i\right) \end{cases}\quad t \geq 0 \tag{4}$$

They differ only in amplitude, by factors depending only on τ_i/T. The ratios of responses (ii) and (iii) to (i) are plotted in Figure 1 for a range of τ_i/T.

The large ratios for time constants comparable with T imply high uncertainty in the quick-flow response to rainfall in any interval, due solely to ignorance of its time course. Although calibrating a model averages the responses over the record, the model's performance on each large flow peak, as well as overall, may well be of concern, and Figure 1 implies high uncertainty for individual peaks. The effect of averaging in calibration can be analysed by assuming that, in finding h_k for given lag kT, the rain

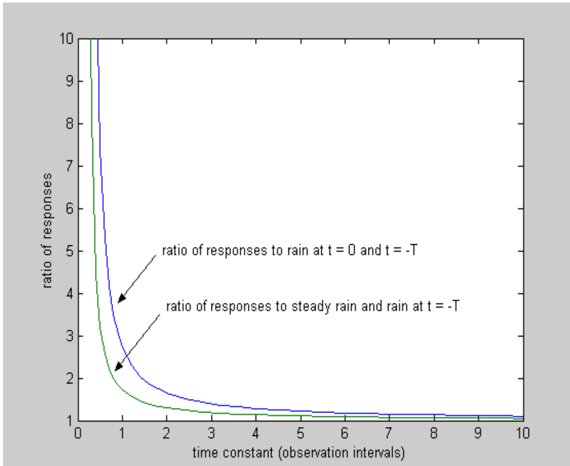

Figure 1. Ratios of amplitudes of exponential flow-response components due to various rainfall histories over $-T < t \le 0$

intensity in the time step ending kT ago is uniform on average over the record. This just says that no particular point between $(k+1)T$ and kT ago has different average rain intensity from any other; it says nothing about the shape or duration of rainfall events. The mean response to rain in the time step ending t ago is the response at time t to the mean rain, since expectation and integration are linear operations:

$$E[q(t)] = E\left[\int_{-T+}^{0} u(\tau)h(t-\tau)d\tau\right] = \int_{-T+}^{0} E[u(\tau)]h(t-\tau)d\tau \quad (5)$$

so (iii) of (4) also gives the mean response to rainfall not favouring any particular point in the time step.

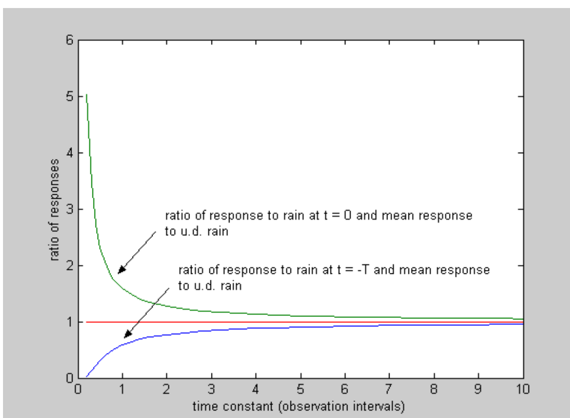

Figure 2. Ratios of responses to concentrated rainfall and mean response

Figure 2 replots (4) with this mean response as reference, to show the variability of unit-rainfall response about the mean. The variability is large. For example, if $\tau_i = T$ the response to rainfall in any one time step may approach 58% higher or 42% lower than the mean. Even with $\tau_i = 2T$ it may approach 27% higher or 23% lower.

2.3 Error due to uncertainty in delay of start of response

Flow response also incurs uncertainty from uncertainty in the dead time (pure delay) between rainfall and the start of the response to it. Rainfall-flow models usually take this delay as constant, yet transport delay through catchments varies with flow. Indeed, event-by-event variation of dead time with rainfall intensity is often visible in rainfall-flow records. The delay registered also varies with the pattern and movement of rainfall with respect to the rain gauge(s). A discrete-time model can be made responsive to varying dead time by allowing parameters to time-vary (Norton [1975]), but modelling of all influences on dead time would be complicated and difficult even with enough rainfall observations. The extent of error due to ignoring variation of dead time is readily analysed, however.

As before, consider an exponential IUH component, now subject to dead time t_d, so that

$$h_i(t) = g_i \exp(-(t-t_d/\tau_i)) = g_i \exp(t_d/\tau_i)\exp(-t/\tau_i),\ t \ge t_d \quad (6)$$

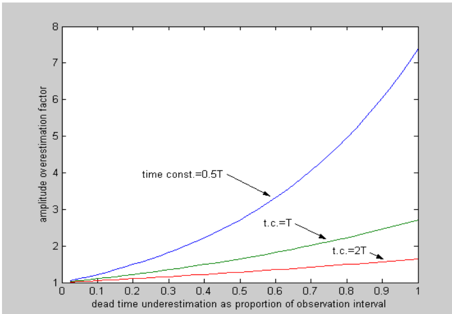

Figure 3. Overestimation of exponential response component due to underestimating dead time

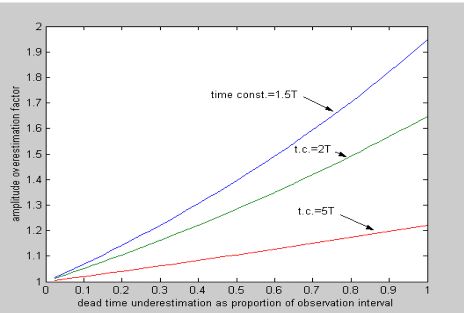

Figure 4. Overestimation for slower response components

Thus dead time just alters the amplitude of the exponential at any instant during it. Underestimating the dead time by δt_d causes the amplitude to be overestimated by a factor $\exp(\delta t_d / \tau_i)$, plotted for positive δt_d as a function of $\delta t_d / T$ for various τ_i / T in Figure 3.

The errors due to modest errors in dead time can be large. They depend strongly on the time constant of the response component, so are most serious for the quick-flow component, but Figure 4 shows that they are still substantial even for slow-flow components with time constants considerably greater than the observation interval.

2.4 Errors in quick-flow slow-flow model due to transformation from linear model

An effective-rainfall flow model is often obtained initially as an input-output equation linear in its parameters and variables, *e.g.* an ARMAX model (Jakeman *et al*. [1990]; Ljung [1995]). Linearity greatly simplifies parameter estimation and uncertainty analysis (Norton [1986]), but for physical interpretability the model must be transformed to another form, such as a unit-pulse response with additive quick-flow and slow-flow components. The transformation is easy for a linear second-order model but typically amplifies uncertainties. Denoting by q_k, u_k and e_k the flow, effective rainfall and equation error at time $t = kT$, the pseudo-regression

$$q_k = -a_1 q_{k-1} - a_2 q_{k-2} + b_0 u_k + b_1 u_{k-1} + e_k \tag{7}$$

can be rewritten in z-transform form to give the input-output relation

$$Q(z^{-1}) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} U(z^{-1}) \equiv \left(\frac{g_1}{1 - p_1 z^{-1}} + \frac{g_2}{1 - p_2 z^{-1}} \right) U(z^{-1}) \tag{8}$$

Here the transfer function linking the effective rainfall and flow transforms $U(z^{-1})$, $Q(z^{-1})$ has been split into partial fractions whose inverse z-transforms give sampled exponential components $g_1 p_1^k$ and $g_2 p_2^k$, $k \geq 0$, of the unit-pulse response, with time constants $\tau_1 = -T / \ln p_1$, $\tau_2 = -T / \ln p_2$. The transformation is algebraically simple:

$$p_1 = \left(-a_1 + \sqrt{a_1^2 - 4a_2} \right) / 2,\ p_2 = \left(-a_1 - \sqrt{a_1^2 - 4a_2} \right) / 2,\ g_1 = \frac{p_1 b_0 + b_1}{p_1 - p_2},\ g_2 = \frac{p_2 b_0 + b_1}{p_2 - p_1} \tag{9}$$

but its effects on the uncertainties in the parameters of (7) must be examined. The local normalized sensitivities *(proportional change in factor)/(proportional change in result)* of the p's and g's to the a's and b's are

$$S_{a_1}^{p_1} \equiv \frac{a_1}{p_1}\frac{\partial p_1}{\partial a_1} = \frac{p_1 + p_2}{p_1 - p_2},\ S_{a_1}^{p_2} = \frac{p_1 + p_2}{p_2 - p_1},$$
$$S_{a_1}^{g_1} = \frac{(p_1 + p_2)(p_1 g_2 - p_2 g_1)}{(p_1 - p_2)^2 g_1},\ S_{a_1}^{g_2} = \frac{(p_1 + p_2)(p_2 g_1 - p_1 g_2)}{(p_1 - p_2)^2 g_2},$$

$$S_{a_2}^{p_1} = \frac{p_2}{p_2 - p_1},\ S_{a_2}^{p_2} = \frac{p_1}{p_1 - p_2},\ S_{a_2}^{g_1} = \frac{p_1 p_2 (g_1 - g_2)}{(p_1 - p_2)^2 g_1},\ S_{a_2}^{g_2} = \frac{p_1 p_2 (g_2 - g_1)}{(p_1 - p_2)^2 g_2},$$

$$S_{b_0}^{g_1} = \frac{p_1 (g_1 + g_2)}{(p_1 - p_2) g_1},\ S_{b_0}^{g_2} = \frac{p_2 (g_1 + g_2)}{(p_2 - p_1) g_2},\ S_{b_1}^{g_1} = \frac{p_2 g_1 + p_1 g_2}{(p_2 - p_1) g_1},\ S_{b_1}^{g_2} = \frac{p_2 g_1 + p_1 g_2}{(p_1 - p_2) g_2} \qquad (10)$$

A numerical example will show that some of these sensitivities are likely to be high. For $p_1 = 0.5,\ p_2 = 0.9,\ g_1 = 3g_2$, *i.e.* time constants of 1.44 and 9.49 days and a quick-flow component three times the size of the slow-flow component, (10) gives

$S_{a_1}^{p_1} = -3.5, S_{a_1}^{p_2} = 3.5, S_{a_1}^{g_1} = -6.42, S_{a_1}^{g_2} = 19.25$, $S_{a_2}^{p_1} = 2.25, S_{a_2}^{p_2} = -1.25, S_{a_2}^{g_1} = 1.875, S_{a_2}^{g_2} = -5.625$,

$S_{b_0}^{g_1} = -1.667, S_{b_0}^{g_2} = 9, S_{b_1}^{g_1} = 2.667, S_{b_1}^{g_2} = -8$. A few percent uncertainty in a_1 or about 10% uncertainty in a_2, b_0 or b_1 is enough to make the estimate of slow-flow amplitude unusable. The quick-flow amplitude also is very sensitive to uncertainty in a_1. Indeed, every normalized sensitivity is greater than unity in size, so uncertainty is amplified in every relation in computing the partial fractions. The numbers in this example are not contrived; the high sensitivities found here are typical.

It is also easy to show that the normalized sensitivity of τ to p is τ/T, so the estimated slow-flow time constant, particularly, is highly sensitive to uncertainty in the corresponding pole. However, this is less serious than it seems, as the slow pole is normally well defined by the flow record in catchments displaying clear quick- and slow-flow components.

3 MEASURING MODEL PERFORMANCE

3.1 Alternative measures of model fit

The fit of the output of a hydrological model to the observed output is most often measured by the Nash-Sutcliffe efficiency (coefficient of determination)

$$R^2 = 1 - \sum_{t=1}^{N} (y_t - \hat{y}_t)^2 / \sum_{t=1}^{N} (y_t - \hat{\bar{y}})^2 \qquad (11)$$

where t indicates time, y the observed output with sample mean $\hat{\bar{y}}$, and $\hat{y}_t$ the model output. It is commonly interpreted as the proportion by which the output mean-square error (MSE) of the model reduces the sample variance of y. Equally it can be viewed as comparing the model output MSE with that of the crude estimator $\hat{y}_t = \hat{\bar{y}}$. Usually R^2 measures the performance of a model driven by observed input alone, with any past output required supplied by earlier model-output values rather than observations, *i.e.* with the model run in simulation mode. The model could be compared with any other input-independent output estimator. If some such estimator performs better than $\hat{y}_t = \hat{\bar{y}}$, its performance is a more stringent benchmark. Specifically, consider an input-independent predictor of the output k time steps ahead. For fair comparison, k must conform with the aim of the model. If the model is to predict over l steps, k should equal l. On the other hand, if the model aims to explain the output on the sole basis of past observed input (as in simulation mode), one can ask how much better the output is explained by the input, *via* the model, than by the earlier output *via* a k-step predictor. The comparison most favouring the model is with k large enough for past output to be uninformative. The least favourable is with a 1-step predictor.

A simple, k-step, output-to-output predictor, linear in its parameters, can be found by minimizing the mean squared error

$$s = E\left[(y_t - \hat{y}_t)^2\right] \tag{12}$$

by choice of the parameter vector **a** of the predictor

$$\hat{y}_{tk} = \mathbf{a}^T \mathbf{f}_{t-k} \tag{13}$$

where $\hat{y}_{tk}$ predicts y_t from time t-k, $\mathbf{a}^T \equiv \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_m \end{bmatrix} \equiv \begin{bmatrix} a_0 & \mathbf{a}'^T \end{bmatrix}$, $\mathbf{f}'_{t-k}$ comprises any collection of m functions of observed output values no later than time t-k and $\mathbf{f}_{t-k} \equiv \begin{bmatrix} 1 & \mathbf{f}'^T_{t-k} \end{bmatrix}^T$. The predictor includes a constant term $a_0 \times 1$ to cover any long-term component not dependent on recent output behaviour. For a stationary point of s,

$$\frac{\partial s}{\partial \mathbf{a}} = 2E\left[\mathbf{f}_{t-k}(\mathbf{a}^T \mathbf{f}_{t-k} - y_t)\right] = \mathbf{0} \tag{14}$$

so the minimum-mean-square-error (MMSE) estimator is

$$\mathbf{a} = E\left[\mathbf{f}_{t-k}\mathbf{f}^T_{t-k}\right]^{-1} E\left[\mathbf{f}_{t-k} y_t\right] \tag{15}$$

providing the inverse exists. It does if $E\left[\mathbf{f}_{t-k}\mathbf{f}^T_{t-k}\right]$ is positive-definite, which is so unless $\boldsymbol{\alpha}^T E\left[\mathbf{f}_{t-k}\mathbf{f}^T_{t-k}\right]\boldsymbol{\alpha} \equiv E\left[\left(\boldsymbol{\alpha}^T \mathbf{f}_{t-k}\right)^2\right] = 0$ for some non-zero $\boldsymbol{\alpha}$, implying that $\boldsymbol{\alpha}^T \mathbf{f}_{t-k} = 0$ in every realisation, clearly not true. As the Hessian of s with respect to **a** is $2E\left[\mathbf{f}_{t-k}\mathbf{f}^T_{t-k}\right]$, the positive-definiteness confirms that (15) minimizes s. Parameter vector **a** is constant if the output series is stationary, which seems dubious for river flow but is justified by the expectation in (12) being over an indefinitely long flow sequence, averaging out seasonal and other variations. This conforms with R^2 measuring fit over the entire record.

Given the definition (12) of s, it is no surprise that the MMSE estimator (15) is the probabilistic counterpart of the ordinary least-squares estimate of **a**. Now

$$E\left[\mathbf{f}_{t-k} y_t\right] = E\left[\left(\mathbf{f}_{t-k} - \begin{bmatrix} 1 \\ \bar{\mathbf{f}} \end{bmatrix} + \begin{bmatrix} 1 \\ \bar{\mathbf{f}} \end{bmatrix}\right)(y_t - \bar{y} + \bar{y})\right] = \begin{bmatrix} \bar{y} \\ \sigma^2 \mathbf{r}_{t-k} + \bar{\mathbf{f}}\bar{y} \end{bmatrix} \tag{16}$$

$$E\left[\mathbf{f}_{t-k}\mathbf{f}^T_{t-k}\right] = E\left[(\mathbf{f}_{t-k} - \begin{bmatrix} 1 \\ \bar{\mathbf{f}} \end{bmatrix} + \begin{bmatrix} 1 \\ \bar{\mathbf{f}} \end{bmatrix})(\mathbf{f}_{t-k} - \begin{bmatrix} 1 \\ \bar{\mathbf{f}} \end{bmatrix} + \begin{bmatrix} 1 \\ \bar{\mathbf{f}} \end{bmatrix})^T\right] = \begin{bmatrix} 1 & \bar{\mathbf{f}}^T \\ \bar{\mathbf{f}} & \sigma^2 \mathbf{R}_{t-k} + \bar{\mathbf{f}}\bar{\mathbf{f}}^T \end{bmatrix} \tag{17}$$

with $\bar{y}$ the mean of the output, σ its standard deviation, $\bar{\mathbf{f}}$ the mean of $\mathbf{f}_{t-k}$, $\mathbf{r}_{t-k}$ an m-vector with $E\left[f_{t-k,i}\, y_t / (\sigma_i \sigma)\right]$ as element i, σ_i the standard deviation of $f_{t-k,i}$, the ith of the functions making up $\mathbf{f}_{t-k}$. $\mathbf{R}_{t-k}$ has $E\left[f_{t-k,i}\, f_{t-k,j} / (\sigma_i \sigma_j)\right]$ as element (i,j). Substituting (16) and (17) into (14) gives

$$\left.\begin{aligned} a_0 &= \bar{y} - \bar{\mathbf{f}}^T \mathbf{a}' \\ \bar{\mathbf{f}}(\bar{\mathbf{y}} - \bar{\mathbf{f}}^T \mathbf{a}') + (\sigma^2 \mathbf{R}_{t-k} + \bar{\mathbf{f}}\bar{\mathbf{f}}^T)\mathbf{a}' &= \sigma^2 \mathbf{r}_{t-k} + \bar{\mathbf{f}}\bar{y} \end{aligned}\right\} \tag{18}$$

From the second equation of (18),

$$\mathbf{a}' = \mathbf{R}_{t-k}^{-1}\mathbf{r}_{t-k} \tag{19}$$

In practice, $\bar{y}$, $\mathbf{R}_{t-k}$ and $\mathbf{r}_{t-k}$ are replaced by unbiased estimates based on the output record. The simplest special case of this predictor is when $\mathbf{f}_{t-k}$ consists merely of y_{t-k}, giving $\mathbf{R}_{t-k} = 1$, $\mathbf{r}_{t-k} = r_k$, $a_1 = r_k$, $a_0 = \bar{y}(1-r_k)$ with r_k the output autocorrelation at lag k, so using sample mean and lag-k autocorrelation of the output,

$$\hat{y}_t = \hat{\bar{y}} + \hat{r}_k(y_{t-k} - \hat{\bar{y}}) \tag{20}$$

This predictor takes an initial observation-free estimate $\hat{\bar{y}}$ and corrects it by a term proportional to observed error $y_{t-k} - \hat{\bar{y}}$ and weighted by the correlation between y_{t-k} and y_t. By contrast, the estimator $\hat{y}_t = \hat{\bar{y}}$ implicit in R^2 makes no correction and effectively takes the autocorrelation as zero. The output MSE corresponding to (20) is

$$E\left[(y_t - \hat{y}_{tk})^2\right] = E\left[(y_t - \bar{y} - r_k(y_{t-k} - \bar{y})^2\right] = (1 - r_k^2)\sigma^2 \tag{21}$$

As $E\left[(y_t - \bar{y})^2\right]^2$ is σ^2 by definition, if R_k^2 denotes the statistic comparing the model MSE with that of the k-step predictor (20), its theoretical value is

$$R_k^2 = 1 - \frac{E\left[(y_t - \hat{y}_t)^2\right]}{E\left[(y_t - \hat{y}_{tk})^2\right]} = 1 - \frac{1-R^2}{1-r_k^2} = R^2 - \frac{r_k^2(1-R^2)}{1-r_k^2} < R^2 \tag{22}$$

3.2 Practical examples

The differences between R^2 and the measures of model fit based on MMSE k-step output-to-output predictors will be illustrated on roughly 30-year records of daily flow in contrasting catchments: at Tinderry below 506 km^2 of the Googong catchment and at Gingera below 148 km^2 of the Cotter catchment, both near Canberra, ACT, Australia. The former catchment is largely grazing or low-intensity forestry with some farm dams, and the latter is higher, steeper and near-natural.

Figures 5 and 6 show the sample autocorrelation functions of these flow records. The autocorrelation is strong (> 0.4) at lags up to 2 days for Tinderry and 15 or so days for Gingera. The more prominent slow flow component of the latter, due to its greater natural

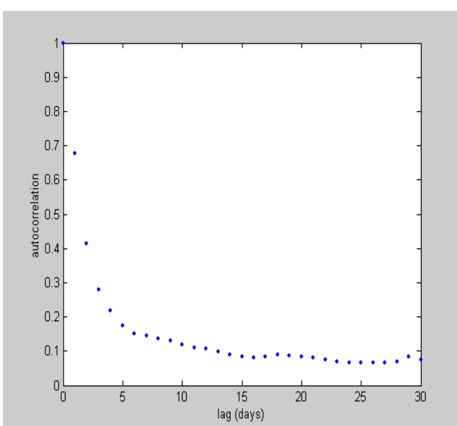

Figure 5. Sample autocorrelation function of daily flow records at Tinderry

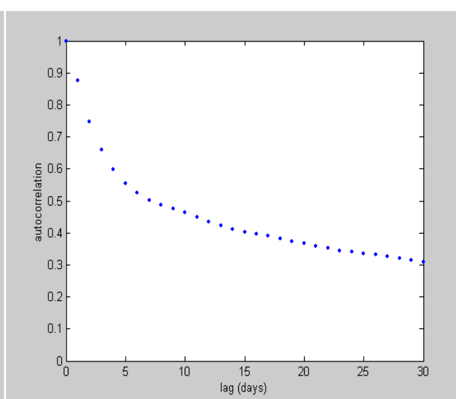

Figure 6. Sample autocorrelation function of daily flow records at Gingera

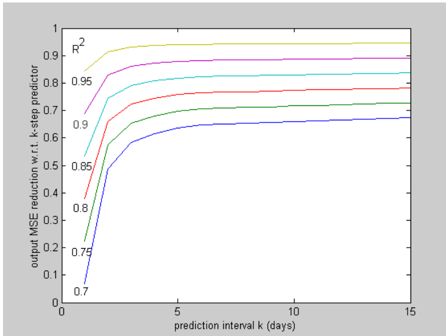

Figure 7. R_k^2 as a function of prediction interval k and R^2 for Tinderry

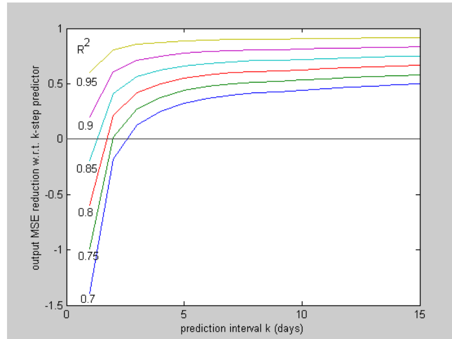

Figure 8. R_k^2 as a function of prediction interval k and R^2 for Gingera

storage capacity, accounts for the difference. Figures 7 and 8 display the resulting measure of fit R_k^2 given by (22) as a function of prediction interval k, for a range of values of R^2. R_k^2 tests how much of the output variation left by a very simple output-to-output predictor is removed by the model. The discrepancy between R_k^2 and R^2 is quite small for the Tinderry record except for k = 1 or 2, but for the Gingera record it is large at k = 5 and notable even at k = 15. Thus for some catchments crude flow predictors of the form (20) explain enough of the flow variation to show a given model much less favourably than does R^2.

4 CONCLUSIONS

Uncertainties in the instantaneous unit hydrograph due to ignorance of detailed rainfall history and rainfall-flow dead time, and conversion from a linear-in-parameters to a quick-flow slow-flow model, have been examined. They may be large unless the flow sampling interval is very short compared with the response time constants .

A family of model-performance measures analogous to Nash-Sutcliffe efficiency but based on simple k-step predictors of flow has been presented. They offer an alternative to R^2 whenever comparison with a predictor using earlier flow, but not rainfall, is permitted.

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