Modelling Connectivity Between Pollutant Source Areas & Streams

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Abstract: The nature of pollutant connectivity between unsealed forest roads and adjacent nearby streams is examined numerically in terms of spatial and temporal patterns of runoff generation, erosion, and sediment transport. In this paper we consider the relative effects of rainfall intensity and duration, surface roughness, infiltration rate, macropore flow and sediment detachment and transport. The objective of the numerical simulation of this hillslope system is to identify the dominant processes and parameters that affect the degree of pollutant connectivity between roads and streams. The St. Venant equations are applied to extract a two-dimensional diffusion wave model with variable conductivity and diffusivity to represent the behaviour of flow dynamics mathematically. The resulting non-linear partial differential equation was solved with appropriate initial and boundary conditions using the MacCormack finite difference numerical technique. Sediment detachment due to rainfall and flow dynamics is linked to the equations for mass conservation and continuity to represent erosion and was solved numerically using the finite difference method. The current numeric representation involves numerous parameters, many of which are unavailable for practical field application. Therefore, present work deals with the selection and/or development of methods to determine the relative significance of each process or parameter, so as to ultimately reduce the parameter space of this complex hillslope problem, improving our ability to scale-up the impacts of forest roads on catchment water quality in future works.

Keywords: Two-Dimensional Overland Flow; Sediment Transport Modelling; MacCormack Finite Difference Scheme; Parsimonious Model

1. INTRODUCTION

Nowadays it has been well understood through the work of authors such as Anderson et al. [1998], Ziegler et al. [2000] and Motha et al. [2004] that unsealed forest roads deliver considerable amount of sediment to water receiving bodies which adversely affects water quality and spawning habitats. The concept of hydrological and sedimentological connectivity has been examined and evaluated by various hydrologists and geomorphologists around the world eg. (Sivapalan et al. [2006], Lane, SN et al. [2007], Roulet et al. [2007] and Takken et al. [2008]). Often research is focused at the watershed scale and does not solely consider the physical concepts of water and soil interactions. Moreover, the conclusions are often based on the findings reached through the formation of empirical models. Other researchers have taken advantage of physical process modelling, eg. (Zhang et al. [1989], Tayfur et al. [1993], Lal [1998], Leonard et al. [2001], Miyamoto et al. [2006] and Xu et al. [2007]) using the physics and dynamics of flow characteristics to further our understanding of soil erosion and connectivity between sediment sources and streams. Often this is achieved through the mathematical representation of overland flow
using the Saint Venant equations. These works have all been extremely useful in discovering true nature of runoff generation and sediment transport in a system comprised of unsealed forest roads and its nearby hillslope and stream. Moreover, these two different approaches, either empirical or numerical, have each their own advantages and drawbacks in understanding hydrological concepts. Therefore, the key flow and erosion processes that impact water quality in such systems still remain uncertain among hydrologists. This paper is written to link these diverse empirical and numerical strategies and propose an approach to discover a simpler more acceptable model structure that takes advantage of both methods. The new model is parsimonious and captures the most significant processes involved, while retaining the support of the underlying physical and dynamic laws of nature.

2. GOVERNING EQUATIONS

2.1 Flow

The shallow water St. Venant equations are commonly used to describe overland flow. The full St. Venant equations describe the process of overland flow very well, however the system of equations to be solved rapidly becomes large and due to the computational requirements the application of these equations becomes inefficient. Simplified routing methods are available with fewer parameters and these have significant advantages in terms of feasibility and practicality. Singh, VP et al. [2005] investigated the errors of kinematic and diffusive wave approximation of St. Venant equations and suggested that the diffusive wave approximation is in excellent agreement with the full St. Venant equations. The approach neglects the local and convective accelerations from the St. Venant equations and utilizes the continuity equation to form the governing equations for the model. The system is comprised of two parts; the road, and the hillslope. The former will be examined in one-dimensional form while the latter, because of its importance and level of dispersion, is evaluated in two dimensions. The continuity and momentum equations for runoff from the unsealed forest road are described in 1D in the following equations:

\[
\frac{\partial h}{\partial t} + \frac{\partial (ah)}{\partial x} = i_x \tag{1}
\]

\[
S_f = S_{0x} - \frac{\partial h}{\partial x} \tag{2}
\]

Whereas for the hillslope part of the model (in 2D) these equations will change to the following:

\[
\frac{\partial h}{\partial t} + \frac{\partial (ah)}{\partial x} + \frac{\partial (vh)}{\partial y} = i_x - \frac{\partial q_y}{\partial x} \frac{\partial h}{\partial y} \tag{3}
\]

\[
S_{fx} = S_{0x} + \frac{\partial h}{\partial x} \tag{4}
\]

\[
S_{fy} = S_{0y} - \frac{\partial h}{\partial y} \tag{5}
\]

The parameters in equations (1) through (5) are introduced below. The term \(i_x\) in equation (3) refers to excess infiltration or effective rainfall. Bed slopes in x and y directions are shown by \(S_{0x}\) and \(S_{0y}\), while the friction slopes \(S_{fx}\) and \(S_{fy}\) are defined as:

\[
S_{fx} = S_{fxbed} - S_{fxsurf} \tag{6}
\]

\[
S_{fy} = S_{fybed} - S_{fyurf} \tag{7}
\]

where \(S_{fxbed}\), \(S_{fybed}\), \(S_{fxsurf}\) and \(S_{fysurf}\) are bed friction and surface frictions in x and y directions, respectively. The reason for this interpretation is that there is a necessity to include the shear stress from the free water surface to count for the impact of wind and rain splash. According to the work done by Zhang et al. [1989], although the impact of wind in overland flow may be negligible the effect of rain splash is definitely something to be considered. Raindrops fall into the water and this process would cause turbulence and
energy loss in the water surface and increases flow resistance. The bed friction is defined through the Manning equation as follows:

\[ S_{\text{bed}} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^\frac{3}{2}} \] (8)

\[ S_{\text{bed}} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^\frac{3}{2}} \] (9)

where, \(u\) and \(v\) are the components of flow velocity in \(x\) and \(y\) direction, \(n\) is the manning roughness coefficient, and \(h\) is water depth. The surface friction accounts for turbulence as a result of raindrops and has been defined by Zhang et al. [1989] in equations (10) and (11), where \(R\) is the rainfall intensity and \(g\) is the gravitational acceleration:

\[ S_{\text{surf}} = -R \frac{u}{gh} \] (10)

\[ S_{\text{surf}} = -R \frac{v}{gh} \] (11)

The flow velocity in one direction is simply described by Manning’s equation. In two dimensions, the work of Akan [1981], Lal [1998] and Miyamoto et al. [2006] defines the flow velocity components \(u\) and \(v\) as:

\[ u = \frac{1}{n} h^\frac{3}{2} \frac{S_n}{\sqrt{S_f}} \] (12)

\[ v = \frac{1}{n} h^\frac{3}{2} \frac{S_m}{\sqrt{S_f}} \] (13)

\[ S_f = \sqrt{S_n + S_m} \] (14)

The macropore flow which is introduced as \(q_m\) in continuity equation (3) is considered due to its significant role in infiltration through earthworm holes and decayed root channels, which are commonly observed in vegetative areas but not on roads and is therefore not included in equation (1). Weiler et al. [2003] reviewed other works related to macropore density and showed that the surveyed macropore density varies between 45 and 700 per m² depending on vegetation, climate, soil management, etc. These authors illustrated the significance of macropore flow by estimating a range of flow rate between 360-2520 mm/hr for an even low 100/m² macropore density. This range is far more than typical natural rainfall intensity. Despite its relative importance, only few researchers have included this process in their overland flow models. The most important work has been accomplished by Léonard et al. [2004]. These authors assumed macropores have cylindrical shape and the flow through them was then simulated as following equation:

\[ q_m = r \int_C (hUm) \, d\theta \] (15)

where, \(r\) is the macropore’s radius, \(h\) is the flow depth, \(U\) is the two directional velocity, \(C\) is the curve which delimits the boundary of the macropore and \(m\) is the vector perpendicular to the macropore circumference. When assuming uni-directional flow toward the centre of the macropores, equation (15), would change to the following equation:

\[ q_m = 2 rhU \] (16)

Ahuja [1993] used Poiseuille’s law and by assuming gravity flow (unit hydraulic head gradient) suggested the following equation for the maximum flow capacity of macropores per unit area of the soil:

\[ K_{\text{max}} = \frac{N \rho g \pi r^4}{8 \mu} \] (17)
where, $K_{\text{max}}$ is the maximum flow capacity, $N$ is the number of macropores per unit area, and $\mu$ is the dynamic viscosity of water. Therefore, flow through macropores is always controlled by the value of $K_{\text{max}}$.

### 2.2 Sediment

Sediment transport is modelled through the mass continuity equations that would link flow characteristics such as water depth and velocity to the sources of erosion on land. Erosion itself is mainly due to two important mechanisms of rain splash detachment and flow detachment. Lane, LJ et al. [1988] and Woolhiser et al. [1990], used the continuity of sediment mass to develop a model that would describe the two-dimensional erosion process in overland flow based on either the transport capacity or the sediment availability in the form of equation (18). Note that for the one-dimensional analysis on the road surface the term reflecting the $y$ direction will be excluded.

$$
\frac{\partial hc}{\partial t} + \frac{\partial (uhc)}{\partial x} + \frac{\partial (vhc)}{\partial y} = \frac{1}{\rho_s} (D_{sd} + D_{df}) \quad (18)
$$

where, $c$ is the sediment concentration by volume, $\rho_s$ is the sediment particle density, $D_{rd}$ is the detachment rate due to rainfall splash and $D_{fd}$ is the detachment/deposition rate due to overland flow. The latter two parameters are described in the work of Foster et al. [1995] and Tayfur [2001] by following equations:

$$
D_{sd} = \alpha R^\beta \quad (19)
$$

Detachment ($q_s < T_c$)

$$
D_{sd} = k_r (\tau - \tau_c) (1 - \frac{q_s}{q_c}) \quad (20)
$$

Deposition ($q_s > T_c$)

$$
D_{df} = 0.5 V_f (T_c - q_s) \quad (21)
$$

where, $k_r$ is the flow erodibility parameter in $\text{s/m}$, $R$ is the rainfall intensity in $\text{mm/hr}$. $\alpha$ and $\beta$ are the soil detachability coefficient and exponent, respectively and their ranges, according to Sharma [1993], are 0.00012-0.0086 $\text{kg/m}^2/\text{mm}$ and 1.09-1.44. $q_s$ is the sediment discharge in the flow direction, which is described by the equation below:

$$
q_s = c \rho_s q_s \quad (22)
$$

$V_f$ is particle settling velocity, which its definition and calculation is available in the work of Woolhiser et al. [1990] and $q$ is the sum of total flow per unit width in $x$ and $y$ directions. $T_c$ is the transport capacity of overland flow which is the maximum flux of sediment that flow is capable to transport. It is a function of soil erodibility and particle characteristics and is shown in form of equation (23):

$$
T_c = \eta (\tau - \tau_c)^{k_f} \quad (23)
$$

where, $\eta$ is the sediment erodibility and Tayfur [2001], used the work of (Foster et al, 1982) and applied 0.6 for this variable in his work. $k_f$ is a constant which varies between 1 and 2 and after (Foster et al, 1982), Tayfur [2001], used 1.5 for this constant. $\tau$ is the shear stress which is defined as:

$$
\tau = \frac{\gamma h S}{k} \quad (24)
$$

where, $\gamma$ is the specific density, $h$ is the flow depth and $S$ is the bed slope. $\tau_c$ is the critical shear stress, which describes soil resistance against erosion and is defined as:

$$
\tau_c = \frac{\delta (\gamma_s - \gamma) d}{k} \quad (25)
$$
where, $\delta$ is a constant depending on flow condition. After (Gessler, 1965), Tayfur [2001] used the value of 0.047 for this constant in his work. $\gamma_s$ is the specific density of the particle and $d$ is the size of particle diameter.

3. NUMERICAL MODEL

3.1 Method, initial & boundary conditions

Due to its simplicity and robustness many hydrologists (eg. Hon et al. [2004], Miyamoto et al. [2006], Luo [2007], Xu et al. [2007] and Yoshikawa et al. [2010]) have applied the finite difference method to simulate overland flow. The idea beyond finite difference method is to substitute the partial derivatives of the parameter by its difference quotient approximations and solve the resulting system of algebraic equations either explicitly or implicitly. Various finite difference schemes over the years have been developed but perhaps one of the most reliable schemes applied to overland flow simulation has been the popular MacCormack scheme after MacCormack [1969, 2003]. Many hydrologists such as Garcia et al. [1986], Zhang et al. [1989], Singh, VP et al. [2005] and Lee et al. [2009] have examined the method’s applicability for overland flow modelling and in all these works, the numerical simulation has shown reasonable results. This method consists of two steps: a predictor and corrector, in which forward and backward discretization are used for the parameters of interest in space and the value for $h$, water depth, in next time step is computed from the average of two steps. Once the water depth in predictor stage is determined within the studied area, then its value will be used in equations (2), (4), (5), (12) and (13) to update the parameters and use the generated values to conclude the results in corrector step. The application of this scheme solves the values for $h$, $u$ and $v$ for the diffusive wave equations. After the resolution of these values, through the same scheme, the mass balance equation for sediment in equation (18), will be solved and the sediment concentration at each time step will be determined.

The values for internal nodes will be determined through the numerical modelling as described above. However the initial and boundary conditions should be introduced in order to initiate the simulation. Kahawita et al. [2000], Deng et al. [2005], Luo [2007] and many others assumed that the flow starts in a dry slope surface. Meanwhile in order to escape the mathematical singularity that would result in case of putting the initial flow depth to zero, Zhang et al. [1989] and Kahawita et al. [2000] supposed that a thin layer of water which does not affect the calculation is considered over the surface. According to Kothyari et al. [1997] it is also assumed that initially flow is not carrying any sediment. The upstream boundary for the forest road is introduced in a way that water depth would be zero for all the times. Tayfur [2001], Jain et al. [2005] and Luo [2007] have set the flow gradient to zero for downstream and this condition is imposed for the sediment transport equation as well. The upstream condition for the second part of the simulation process (hillslope) is set in a manner that is coherent with the outflow hydrograph of the forest road simulation. Parsons et al. [1999] suggested having similar concepts for the downstream boundary condition as the road part with respect to flow and sediment transport.

3.2 Stability and calibration

Similar to all the finite difference schemes, the time step in the MacCormack method is also restricted in a way to satisfy the Courant condition. This condition simply states that flow cannot progress ahead of the grid size within the selected time step. The condition for one-dimensional analysis of MacCormack scheme has been well described by Singh, V et al. [1996] and the two-dimensional analysis for stability issues according to Singh, V et al. [1996] and Toro [1999] is as follows:

$$C_{tx} = \frac{\Delta t * \max(|u| + \sqrt{gh})}{\Delta x}$$

(26)

$$C_{ty} = \frac{\Delta t * \max(|u| + \sqrt{gh})}{\Delta y}$$

(27)
where, $C_{rx}$ and $C_{ry}$ are Courant numbers in $x$ and $y$ directions, respectively and their sum must always be between 0 and 1. The model will be calibrated with real field data to determine the values for its coefficients.

4. PARSIMONIOUS MODEL

4.1 Spatial characteristics of soil, flow & erosion and its influence

The effects of spatial variability in soil hydraulic properties have been investigated by several authors (eg. Brath et al. [2000], Iqbal et al. [2005], Isham et al. [2005] and Huidae et al. [2009]). Woolhiser et al. [1996] used kinematic wave formulation to show that runoff hydrographs are strongly dependent on the trends in hydraulic conductivity. Corradini et al. [1998], illustrated that as random variability of saturated hydraulic conductivity increases, runoff starts sooner and increases slower as opposed to uniform conductivity. Huang et al. [2002], undertook some field and lab experiments and suggested that variability in runoff and sediment production could be attributed to micro topography and surface variations. Despite all these efforts many of the effects of spatial characteristics of flow and soil surface are yet to be discovered. Some of these parameters may include: soil moisture and texture, macropores, surface roughness, topography, particle size of sediment and its characteristics, etc. In order to evaluate their relative effects within the area of interest, one could change the pattern of variability for these parameters in decreasing, increasing and random orders. The strategy toward developing a parsimonious model should take into account these variability patterns and must generate a model that would capture these uncertainties. Therefore, it would be wise to analyse the degree of their importance through some independent simulations in parallel with simulation of the cumulative amount of runoff and sediment at the end of the slope.

4.2 Sensitivity analysis and model generation

Sensitivity analysis is the process of estimating the rate of changes in model output as a result of changes being made in model inputs. In this study, each process will be examined separately to evaluate its role and importance. The NS coefficient introduced by Nash et al. [1970], which is recommended by the American Society of Civil Engineers Task Committee on Definition of Criteria for Evaluation of Watershed Models of the Watershed Management Committee [1993], is proposed here to analyze the measure of model fitness. Through inclusion and exclusion of each process and comparing the results with the numerical model’s outcomes which now act as real data, the necessity for consideration or omission of the process will be evaluated. When it is determined which parameters and processes have the most influence on the output results, they will be set aside for the next stage, which is to form some kind of empirical model that will best fit the simulation results from each event. Two sets of empirical models will be developed, one for prediction of flow and the other for sediment transport. For each event, the relevant model would describe the behaviour of the road and its nearby hillslope in terms of flow and its velocity, water depth, sediment transport characteristics and the distribution of flow and erosion. The mesh grid used for the simulation will be the domain in which the reliability of the generated model will be checked against the simulated outputs.

5. CONCLUSION

In this paper, the significant parameters and processes involved in runoff generation and sediment transport at hillslope scale are discussed. The MacCormack finite difference method is proposed for the solution of two-dimensional diffusive wave equations for the flow routing and sediment transport. Suitable initial and boundary conditions are then examined to solve the numerical model. The proposed strategy will then be applied to the numerical model to discover the role of spatial variability of each parameter. During this process the significant processes and parameters will be determined. The goal is to step up a level to make a parsimonious model out of the most significant variables. Therefore, neither the model would be impractical and inaccurate, nor it would suffer from over
parameterization. The approach toward reaching this objective has been described to better conceptualize the methodology for future works that are to be accomplished.

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