Urban Growth and the Simon Process: the Japanese Case

T. Fukiharu

Faculty of Economics, *Hiroshima University*, *Japan* (fukito@hiroshima-u.ac.jp)

Abstract: It has been known that, worldwide, the distribution of accumulated city population exhibits the Pareto distribution with parameter value 1. Theoretically, the Simon process has been known for the analysis of Pareto distribution. This process is a general growth model of lumps, in which the smallest lump is newly produced with probability π_1, while one of the existing lumps grows to the larger lump with the probability of $(1-\pi_1)$. The comparison between the US data and the Japanese data suggests the modification of the Simon process, since the Japanese data exhibits the concavity of the frontier for the logarithmic transformation of accumulated city population. Thus, the possibility of collapse of the largest city to two smaller cities through the environmental or political reasons is introduced. Historically, the newly formed Japanese governments have occasionally changed the capital cities. With the introduction of the probability, π_2, of the above collapse, the modified two-stage Simon process guarantees the concavity. The analysis on the parameter value 1 is also conducted by simulation approach.

Keywords: *pollution, city size, Simon process, transfer of capital, simulation.*

1. INTRODUCTION

There are two approaches in the urban economics: normative and positive ones. One of the main themes in Japan for the former approach is the transfer of the capital. This theme has been often discussed in Japan and South Korea since Tokyo and Seoul suffer from overpopulation, while the rural areas suffer from depopulation. Some argue for the transfer of the capitals, pointing out the high living costs including land price and atmospheric pollution there. The others argue against it, pointing out that the high living cost reflects the high productivity including informational value there. In Japan, by political reasons, the merging of the depopulated towns is under way. The aim of this paper is not to discuss whether the city size distribution in a country is desirable or not. Rather, the theoretical explanation of the city size distribution is the main concern. Thus, the positive approach is adopted in this paper.

It is well-known that in any country all over the world the city size distribution exhibits the Pareto distribution with parameter value approximately one (Zipf [1949], Rosen and Resnick [1980], and Dobkins and Ioannnides [1996]). On the one hand, purely economic explanation has been attempted (Henderson [1988], Fujita, Krugman and Venables [1999]). On the other hand, somewhat more general (non-economic) explanation has also been attempted (Simon [1955], Ijiri and Simon [1977]). In this paper, the latter approach by Simon is adopted. In Section 2, the comparison of the US city size distribution and the Japanese one is made, explaining the necessity to modify the Simon process. In Section 3, the traditional Simon process is explained, while other examples of Pareto distribution are constructed. In Section 4, the (modified) two-stage Simon process is proposed, and the simulations on this process are conducted. The final section concludes this paper.

2. THE COMPARISON OF THE US CITY SIZE DISTRIBUTION AND THE JAPANESE ONE

Although in any country all over the world the city size distribution exhibits the Pareto distribution, there is a slight difference of the distributional shapes between the US and Japan. In this section, this difference is pointed out.

2.1 The US City Size Distribution

The US city size distribution can be easily constructed utilizing *Statistical Abstracts of the United States 2002.* If the cities with more than 100, 000 population are selected and the set of their populations, city sizes, is denoted as "data US", the data contains 239 cities, where the largest city size is 8,008,000 (New York City) and the smallest one is 100,000, while "data US" contains 153 different elements: city sizes. From this set, the following set, "distribution US", is constructed, which is the set of pairs, $\{S,N\}$, where S is the city size in "data US", and N is the number of city sizes in "data US" at least equal to S. The logarithmic transformation of "distribution US" is depicted in Figure 1.

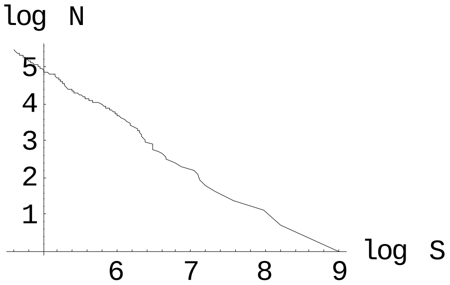

Figure1. The City Size Distribution: US [1]

Utilizing the regression analysis through the least square method (LSM), we have the following functional relation:

$$\log N = 11.6203 - 1.33573 \log S \qquad (1)$$

Here, R^2 value in this regression is quite high: R^2=0.994036.

Zipf [1949] found that for the US city size distribution

$$N = kS^{-\alpha} \qquad (2)$$

holds where α is approximately 1. The distribution of $\{S,N\}$, which satisfies (2) is called the *Pareto distribution*.

2.2 The Japanese City Size Distribution

Rosen and Resnick [1980] found that for any country all over the world the city size distribution exhibits the Pareto distribution with parameter α approximately one. In this subsection, it is examined if the same result obtains for the Japanese case. According to *Japan Statistical Yearbook 2004*, there are 671 cities in Japan. Among the collection of Japanese city sizes, "data J", the largest one is 8,130,408 (Tokyo) and the smallest one is 5,941 (Utashiuchi). In exactly the same way as in 2.1, we construct "distribution J", and its logarithmic transformation is depicted as the *thick* curve in Figure 2. LSM gives rise to the following functional relation:

$$\log N = 17.2024 - 1.03518 \log S \qquad (3)$$

Here, R^2 value in this regression is high: R^2=0.95486.

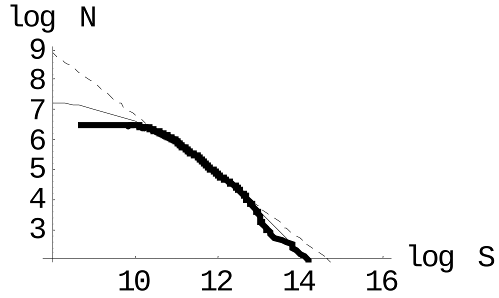

Figure2. The City Size Distribution: Japan [1] and Regression Lines

Between the two results, there is a small difference. The degree of statistical fit for the linear relation in (1) is better than the one in (3). From Figure 2, it appears that there is non-linear functional relation between log S and log N for Japanese case, so that the quadratic LSM is applied to the logarithmic transformation of "distribution J".

In Figure 2, the dashed line is expressed by (3) and the *thin* solid curve depicts the following equation:

$$\log N = -0.626421 + 2.02444 \log S - 0.130256 (\log S)^2 \qquad (4)$$

where R^2 is 0.988933.

Meanwhile, if we construct a subset of "distribution J" by selecting the Japanese cities with population size greater than 100,000, the same procedure gives rise to Figure 3.

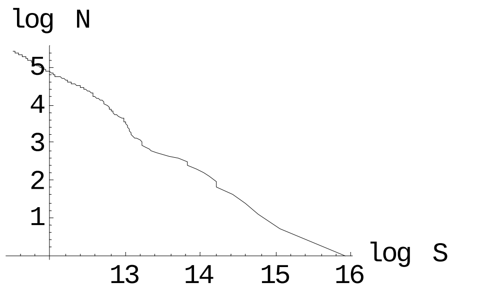

Figure3. The City Size Distribution: Japan [2]

LSM gives rise to the following relation:

$$\log N = 20.6306 - 1.31383 \log S \qquad (5)$$

Here, the R^2 value in this regression is high: R^2=0.988074. Note, however, that it is higher than the one in (3). It may be said that the linearity holds on the functional relation between log S and log N when only the *larger*-sized cities are selected, while the slope is larger (in absolute value).

Returning to the US case, the extended US city size distribution can be easily constructed utilizing *Statistical Abstracts of the United States*

2002, by adding cities with *smaller* population. If the cities with more than 10, 000 population are selected and the set of their populations, city sizes, is denoted as "dataUS1", the set contains 2642 cities.

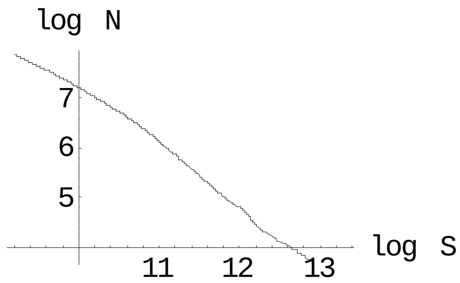

Figure4. The City Size Distribution: US [2]

LSM gives rise to the following functional relation:

$\log N = 18.5346 - 1.13681 \log S$ (6)

Here, the R^2value is high in this regression: $R^2=0.98608$. Note, however, that it is lower than the one in (1).

From the comparison between Figure 1 and Figure 4 and the one between Figure 2 and Figure 3 the following tendency may be observed. The concavity in this tendency was newly found in this paper.

[O1] For both countries, as the data set is enlarged by containing *smaller*-sized cities in the data set, the *absolute value* of estimated parameter for the Pareto distribution becomes *smaller*.
[O2] For both countries, as the data set is enlarged by containing *smaller*-sized cities in the data set, the non-linearity (concavity) of the functional relation between log *S* and log *N* is stronger, while the linearity holds when only the *larger*-sized cities are selected.
[O3] The degree of *concavity* for the functional relation between log *S* and log *N* is stronger in Japan than in the US.

3. SIMON PROCESS

As pointed out in Fujita, Krugman and Venables [1999, Chapter 12], Simon process appears to create the linearity between log *S* and log *N*, where the Simon process is the growing process of cities. This theoretical result, however, is not necessarily compatible with the actual city distribution. As noted in the previous section, the actual city distribution does not necessarily exhibit the linearity of functional relation between log *S* and log *N* over the whole region, but only on the subset of the whole region. This observation of actual city distribution prompts us to modify the Simon process. In this section, the traditional Simon process is explained, while other examples of Pareto distribution are constructed.

3.1. Traditional Simon Process

Simon [1955] constructed a general growing process of "lumps". In what follows, we take "city" as an example of "lump" and regard the dynamic Simon process as the collection of city distributions from the initial period to the last period, T. In this process, as time proceed from current period to the next period, the probability of the creation of a new city (base unit, city size=1) in the next period is given by π_1, and the already existing cities grow in the next period with the probability proportionally to the population. As an example, consider the following special case. Suppose that $\pi_1=0.1$, and currently, there are 8 base unit cities, 2 cities with city size 2, and one city with city size 4. The city size distribution is expressed as {{1,8}, {2,2}, {4,1}}. In this case, the probability of the creation of base unit city in the next period is 0.1, the probability of the expansion of existing base unit city to the city size 2 is 0.9×1×8/(1×8+2×2+4×1), the probability of the expansion of the existing city with city size 2 to the city size 3 is 0.9×2×2/(1×8+2×2+4×1), and the probability of the expansion of the existing city with city size 4 to the city size 5 is 0.9×4×1/(1×8+2×2+4×1). If the new city emerges, the next period's city distribution is {{1,9}, {2,2}, {4,1}}. If the base unit city expands, the next period's city distribution is {{1,7}, {2,3}, {4,1}}, etc. This process continues to the last period T (for the details of the construction of *Mathematica* function for the traditional Simon process, see Fukiharu [2006]).

3.2. Other Examples of Pareto Distribution

Simon process is a very general growing process of "lumps". Simon [1955] pointed out that a lot of distributions, such as the one of the numbers of words used in a book, or the one of the numbers of published papers by researchers in a technical journal, exhibit the Pareto distribution. He proved that if the distribution by Simon process is convergent, then the limit distribution is a Pareto distribution. In this subsection, as another example, the Japanese stock price distribution is examined. In Figure 5, as in the previous section, the set of pairs, {*S*,*N*}, is depicted after logarithmic transformation, where *S* is the stock price in 1988, and *N* is the number of stock prices at least equal to *S*.

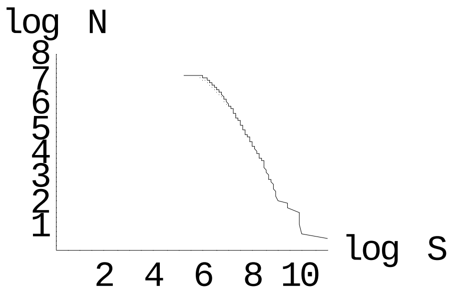

Figure 5. The Japanese Stock Price Distribution in 1988

LSM gives rise to the following functional relation:

log *N*= 15.2277-1.3487 log *S* (7)

Note the R^2 value in this regression is relatively high: R^2 =0.936038. If the stock prices greater than 400 yen but smaller than 8000 are selected, utilizing the regression analysis through LSM we have the following functional relation:

log *N*= 17.4665-1.67308 log *S* (8)

Note that the R^2 value improves to 0.958507, but the slope deviates further from -1, so that [O1] and [O2] in Section 2 hold in this stock price case.

Take another stock price distribution. On December 30, 1998, there were 1597 stock prices in Tokyo Stock Exchange. In exactly the same way as above, by the LSM on this case, we have the following functional relation:

log *N*= 11.2091-0.797215 log *S* (9)

In this case, R^2 is 0.954006. If the stock prices greater than 300 yen, but smaller than 10000 yen, are selected, we have the following relation:

log *N*= 12.7308-1.01585 log *S* (10)

Now the R^2 value improves to 0.993235, and the (absolute) slope increases. Thus [O1] and [O2] in Section 2 hold in this stock price case.

3.3 Reasons for the Non-Linearity

As shown above, in general, the non-linearity holds over the whole region, and the linearity holds on a subset of the whole region, for example, when we include only the larger-sized cities in the data. In this subsection, the reasons for the non-linearity are examined. In the case of stock prices, as the stock price rises due to the excellent corporate performance, the Japanese firms tends to dampen the stock price hike, for example, by the method of stock division. This tendency may put upper limit for the stock prices, creating the bulge on the diagram.

In what follows, the non-linearity in the Japanese city size distribution diagram is examined. As the population of a city expands, the city area also expands to the suburban area due to the shortage of land or environmental reasons, and the suburban areas may become independent cities. This tendency, however, is common to both the US and Japan, and this cannot explain [O3] in Section 2. As one of the factors, which explain [O3], we may refer to the transfers of the capital in the history of Japan.

Japanese people tend to gather around the capital and/or the cities in which the governments were formed. In the history of Japan, the capital of Japan and/or the cities with governments have been transferred so often, that the previous capital dwindles while the new capital is created. The reasons for the transfer vary from the political ones to the environmental ones. As an example of the latter case, the transfer of capital from Nara to Kyoto may be mentioned. In 740, the capital was transferred from Nara to Kuni, from Kuni to Osaka in 744, from Osaka to Shigaraki in 744, from Shigaraki to Nara in 745. Recently, researchers pointed out that the reason for these transfers might well be the air pollution stemming from the construction of huge statute of Buddha. In this decade, the deeply religious Emperor ordered the construction of a great (gilded copper) statute of Buddha at a temple in Nara. According to the researchers the statute was so large that in plating it with gold by the traditional amalgam method a huge amount of mercury was used (50 tons), and the vaporized mercury must have caused severe air pollution (Miwa [1979], Shirasuga [2004]). In 784, the capital was transferred from Nara to Nagaoka, near Kyoto, then, in 794 from Nagaoka to Kyoto.

As examples of political reasons for the transfer of capital, we may mention the rise of *Samurais*, or warriors (for the detail, see Fukiharu [2006]).

4. TWO-STAGE SIMON PROCESS

In this section, following the previous remarks, the two-stage Simon Process is proposed, and utilizing it simulations are conducted.

4.1. Programming of the Two-Stage Simon Process

Suppose that independently of the growth process of the existing cities there exists the probability of the partition of the *largest*-sized city into *small* two cities, π_2, stemming from the reasons referred to in the previous section. Thus, in this section, the two-stage Simon Process is constructed. In subsection 3.1, it was assumed as a specified case that the base-unit city is created with the probability π_1, while one of the existing cities, say, the one with city size i, expands to the city with city size i+1, with the probability $1-\pi_1$. After this stage, suppose that the *largest* city is partitioned into two smaller-sized cities with the probability π_2.

In this subsection, a *Mathematica* command function, simon3 [π_1, π_2, list], is constructed. This function creates the next period's city size distribution, when the current period's city size distribution, "list", and the probabilities: π_1, and π_2, are given arbitrarily (for the details of the construction of this function, see Fukiharu [2006]).

4.2. Simulations of the Two Stage Simon Process

In this subsection, utilizing the command function constructed in the previous subsection, simulations are attempted. First of all, the exercise mentioned in Fujita, Krugman and Venables [1999, Chapter 12] is reproduced. This is the case, in which π_1=0.2 and π_2=0 for our *Mathematica* command function, simon3.

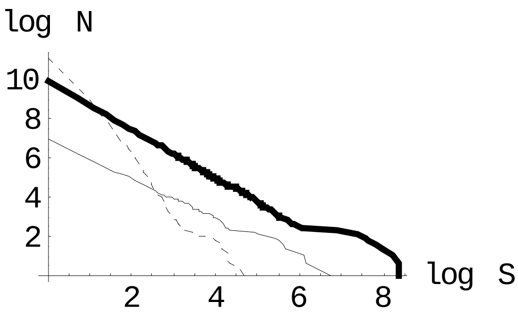

Figure 6. Traditional Simon Process

Setting the initial city size distribution, C[1] as {{1,10}}, by the Simon process: C[t+1]=simon3[0.2, 0, C[t]], t=1,..., T=100,000, we have the final city size distribution, C[T], and its logarithmic transformation is depicted as the *thick* solid curve in Figure 6.

LSM gives rise to the following functional relation:

$$\log N=9.49674-1.11646 \log S \quad (11)$$

The R^2 value in this regression is high: R^2 =0.978927.

Other simulations, which were not conducted in Fujita, Krugman and Venables [1999, Chapter 12], are attempted in this paper. If π_1=0.01 and π_2=0 are assumed, the logarithmic transformation of the final city size distribution is depicted as the *thin* solid curve in Figure 6. LSM gives rise to the following functional relation:

$$\log N=5.98919-0.638409 \log S \quad (12)$$

For (12) R^2 is 0.934146, while the parameter of Pareto distribution is far from -1. If the *smallest* 60 city sizes are selected with extremely large cities deleted, then, we have the following regression result:

$$\log N=6.90396-0.982983 \log S \quad (13)$$

For (13) R^2 is 0.991358.

If π_1=0.70 and π_2=0 are assumed for our command function, simon3, we have the final city size distribution, C[100,000], with 34 elements, and its logarithmic transformation is depicted as the *thin* dashed curve in Figure 6. we have the following functional relation:

$$\log N=10.8689-2.42998 \log S \quad (14)$$

R^2 is 0.966466, while the parameter of Pareto distribution is far from -1. If the *largest* 20 city sizes are selected with small cities deleted, then, we have the following regression result:

$$\log N= 7.50416-1.50796 \log S \quad (15)$$

For (15) R^2 is 0.940985. The conclusions in the simulations so far are as follows.

[S1] As π_1 increases with π_2 set at 0, the overall slope increases.
[S2] With the deletion of the extreme cities, however, the slope comes close to 1 in all cases.

Next, consider the cases with positive π_2. First of all, it is examined if the upper bound on the city size exists when the positive π_2 is assumed. From the simulations with π_1 =0.01and π_2 =0.01, π_1 = 0.7and π_2 =0.01, we may conclude the following.

[S3] When π_2 >0, there exists the upper bound on the city size at final city size distribution.

We might even conclude that the final city size distribution is convergent when π_2>0. Gabaix [1997] examined the convergence of the Simon process when the *largest* city size is fixed at 64.

Next, let us proceed to the *concavity*. Suppose that π_1=0.2 and π_2=0.01. The logarithmic transformation of the final city size distribution is depicted as the *thick* dashed curve in Figure 7, while the *thick* solid curve is the result with π_1 =0.2 and π_2=0.

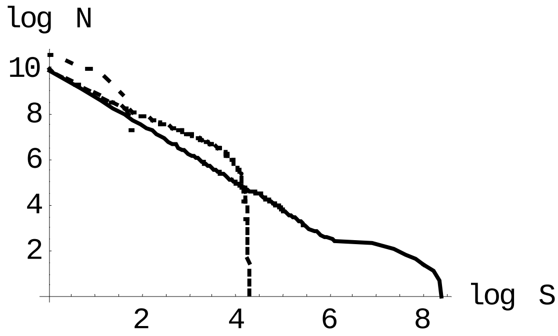

Figure 7. Two-Stage Simon Process

When π_1 =0.2 and π_2=0.2, the final city size distribution is depicted as the dashed curve with a broad interval. In both cases, the concavity appears when π_2>0.

When π_1 is assumed as 0.7 and π_2 varies from 0.01 to 0.2, we have the same result as in Figure 7. Thus, we may conclude the following.

[S4] Compared with the case in which π_2=0, the stronger *concavity* of the functional relation emerges between log *S* and log *N* when π_2>0, while R^2 is lower.

This result may explain [O3], since π_2 for Japanese case may be greater than the US case due to environmental or political reasons.

4. CONCLUSIONS

The aim of this paper is to propose and examine the two-stage Simon process to explain the Japanese city size distribution. In Section 2 of this paper, the difference between the US city size distribution and the Japanese one was pointed out. In Section 3, the Japanese history was discussed, as well as other examples of Pareto distribution in terms of the Japanese stock price distribution. In Section 4, the two-stage Simon process was proposed for the simulations in terms of this command function. The traditional Simon process assumes only π_1: the probability of the emergence of base unit city. The two-stage Simon process assumes π_2: the probability of the partition of the *largest* city, as well as π_1. First of all, assuming π_2=0, simulations of other cases than the one mentioned in Fujita et al. [1999] were conducted, with the conclusion that as π_1 increases the slope of the Pareto distribution increases. Also, it was found that when π_1 is small, the slope is close to 1 when only the *smaller*-sized cities are selected for the data set in the regression, while when π_1 is large, the slope is close to 1 when only the *larger*-sized cities are selected for the data set in the regression. When π_2>0, it was found that the there exists a tendency of the existence of upper bound on the city size. As expected, the functional relation between log *S* and log *N* is *concave* whenπ_2>0. However, the linearity of the functional relation between log *S* and log *N* for a *subset* of the data set was not guaranteed when π_2>0. In this sense, more sophisticated mechanism is required to create the Japanese city size distribution by the Simon process. This task will be pursued in the subsequent papers.

5. References

Cootner, P.H. (ed.). [1964]. *The Random Character of Stock Market Prices*, Cambridge, Mass.: MIT Press.

Dobkins, L.H. and Y.M. Ioannnides. [1996]. "Evolution of the U.S. City Size Distribution." Mimeograph, Tufts University.

Fujita, M, Krugman, P.R., and A.J. Venables. [1999]. *The Spatial Economy: Cities, Regions, and International Trade*, MIT Press,

Fukiharu, T. [2006]. Urban Growth and the Simon Process: the Japanese Case, http://home.hiroshima-u.ac.jp/fukito/index.htm.

Gabaix, X. [1997]. "Zipf's Law for Cities: An Explanation." Mimeograph, Harvard University.

Henderson, J.V. [1988]. *Urban Development: Theory, Fact, and Illusion.* Oxford University Press, Oxford.

Ijiri, Y., and H. Simon. [1977]. *Skew Distributions and the Sizes of Business Firms*. North-Holland, Amsterdam.

Miwa, S. [1979]. "The Great Statue of Nara Buddha from the Viewpoint of Powder Engineering (in Japanese)." *Asahi Graph* (Nov. 16, 1979).Tokyo.

Shirasuga, K. [2004]. "Heijokyo (Nara) Went to Ruin in Mercury Poisoning (in Japanese)." *Nihon Keizai Shimbun* (May 7, 2004), Tokyo.

Rosen, K.T., and M. Resnick. [1980], "The Size Distribution of Cities: An Examination of the Pareto Law and Primacy," *Journal of Urban Economics* 8(2).

Simon, H. [1955], "On a Class of Skew Distribution Functions," *Biometrica* 42.

Zipf, G. [1949]. *Human Behavior and the Principle of Least Effort.* Addison-Wesley, New York.

Acknowledgement: the useful comments by an anonymous reviewer are appreciated.