LPV Approximation of Distributed Parameter Systems in Environmental Modeling

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Abstract: Environmental systems often involve phenomena that are continuous functions not only of time, but also of other independent variables, such as space coordinates. Typical examples are transportation phenomena of mass or energy, such as heat transmission and/or exchange, humidity diffusion or concentration distributions. These systems are intrinsically distributed parameter systems whose description usually requires the introduction of partial differential equations (PDE). Therefore, their modeling can be quite complex, both for what concerns the model construction and its identification. Indeed, a typical approach for the simulation of such systems is the use of finite element techniques. However, this kind of description usually involves a huge number of parameters and requires time-consuming computation while not being suited for identification. For this reason, such models are generally not suitable for control purposes. In many cases, however, the involved phenomena depend on the independent (space) variables in a smooth way, and for fixed values of the independent variables, input/output relations can be satisfactorily represented by linear invariant models. In such conditions, a possible alternative to PDE consists in representing the physical system with a Linear Parameter Varying (LPV) model whose parameters are functions of the independent variables. The advantage of this approach is the relatively simple model obtained, which is directly suitable for control purposes and can be easily identified from input/output data by means of classical techniques. Moreover, optimal identification schemes can be derived for such models, allowing the optimization of the number of measurements. This can be particularly useful in several environmental applications for which the cost of measurements represents a severe constraint. In this paper, the derivation of LPV models for the representation of distributed phenomena in environmental systems is discussed and illustrated with a simulated and a practical example.

Keywords: Distributed parameter systems, Approximation, LPV models.

1 INTRODUCTION

In environmental and agricultural sciences complex systems need often to be described with mathematical models. The development of model structures adequate for practical use is carried on with different approaches, depending on the goals of the modeling process as well as on the available information. Therefore, at one extreme, large simulation models based on the best available knowledge of the involved processes are developed. Such models are often too
complex to be satisfactorily identified from experimental data and too detailed to be suitable for control needs. At the other extreme, simple low order input-output “black box” models are derived only processing available data with no or little use of *a priori* knowledge on the involved process. The choice is therefore between highly detailed models, that reduce the uncertainty in the system representation to a minimum, and “simple” models suitable for control purposes that however imply higher uncertainty in the system representation. Drawbacks of, and alternatives to such approaches are nicely discussed in a recent paper by Young and Chotai [2001].

It is well accepted that models to be used in control applications need to be simple and robust to uncertainties, nevertheless the possibility to incorporate some knowledge on the involved process in the model construction is a desirable feature. Also, the possibility to state a correspondence between the model behavior, its parameters and the physical nature of the real system is often seen as a great advantage.

Most environmental and agricultural processes are intrinsically distributed parameter systems, and their behavior should therefore be described using partial differential equations (PDE) that, besides being function of time, depend also on spatial coordinates. Possible examples are given by processes in which mass or energy transport phenomena occur. The resulting models are infinite dimensional state models that, in general, are difficult to be identified and managed even in the simple linear case.

Indeed, applying finite-differences techniques it is possible to approximate such models by equivalent finite state models driven by ordinary differential equations in which the infinite number of states is replaced by a “large number of states”. Such models are usually presented in discrete-time form to be implemented on digital computers and their large number of states increases for increasing required accuracy in approximating the original infinite dimensional system. The resulting model is therefore suitable for simulations but is far too complex for control purposes, although it has moderate uncertainty.

In control engineering practice high dimensional systems are commonly approximated with lower order linear time-invariant (LTI) models. The case of distributed parameter systems regarded as systems with a “large number of states” is not an exception. Hence, low order approximating models can be derived for distributed parameter systems. It is also possible to add some physical interpretation to the different parameters and features of the approximated model. For example, some of its time constants could be regarded as diffusion constants or transport time constants between the input and the output of the distributed parameter real system. Simplified time-invariant linear parameter models however lose any information about the spatial structure of the original system and cannot account for it although they can be satisfactory from an input-output point of view.

In practice, there are cases in which some information about the spatial structure of the system should be maintained in the simplified model so that results derived for particular settings with respect to the spatial positions of inputs and outputs can be used also when different input-output conditions are of interest.

Hence, some simplified model structure that still preserves some information about the spatial structure of the system is needed. Such structure can be provided by Linear Parameter Varying (LPV) models consisting of a linear lumped parameter model in which the parameters are not constant, but are functions of an extra, possibly vector valued, variable $x$ that can be regarded as an input determining the “operating condition” of the model [Rugh and Shamma [2000], Leith and Leithead [2000], Shamma and Xiong [1999]]. By selecting the $x$ operating condition to be the spatial coordinate of the inputs and/or outputs of the system some information about the spatial structure is included in the simplified model and can be used to derive results valid throughout the system volume.

Remark that this kind of model remains basically a simplified input-output model. It is therefore not suited for replacing the infinite dimensional model driven by partial derivative equations (or its finite element approximation) when simulating the internal behavior of the distributed parameter system.

In this paper, we discuss the use of LPV models for describing distributed parameter systems when information about the spatial structure of the system is needed. In Section 2, the notation
and the main ideas of this approach are developed. In Section 3, a simulated example relative to the determination of the temperature in a bar plunged at its extreme in two fluids at different temperatures is reported. In Section 4, the real case of soil cooling after forced steam heating during a disinfestation process is discussed and the proposed techniques are applied to real data. Finally in Section 5 some conclusions are drawn.

2 LPV APPROXIMATION OF DISTRIBUTED PARAMETER SYSTEMS

Consider a distributed parameter system subject to an input signal $u$. We are interested in the prediction of the forced response of a distributed variable $y(t; x)$ to the input signal $u(t)$ for any time $t$ and spatial coordinate $x$. To this extent we introduce an approximating model which is constituted by the cascade of a delay and an LPV model. Both the delay and the LPV parameters are function of $x$. For the study of such LPV systems with parameter-dependent delays, both for analysis and control, the interested reader is referred to Wu and Grigoriadis [2001]. Moreover continuous time representations are in general replaced with discrete time models.

The resulting model is reported in Figure 1. It follows that the delay block, taking into account transport delays in the process, is defined as follows

$$\tilde{u}(k) = q^{-\Delta(x)}u(k)$$  \hspace{1cm} (1)

where $k$ is (discrete) time, $\Delta(x)$ is an unknown function to be estimated and $q^{-1}$ is the usual unit-delay operator.

The LPV model is based on an autoregressive dynamic structure with exogenous input (ARX)

$$A(q^{-1}; x)y(k; x) = B(q^{-1}; x)\tilde{u}(k) + e(k) ,$$  \hspace{1cm} (2)

where $e(k)$ takes into account measurement and modeling errors and the regressors $A(q^{-1}; x)$ and $B(q^{-1}; x)$ are defined as

$$A(q^{-1}; x) = 1 + a_1(x)q^{-1} + a_2(x)q^{-2} + \ldots + a_{na}(x)q^{-na}$$

$$B(q^{-1}; x) = b_0(x) + b_1(x)q^{-1} + b_2(x)q^{-2} + \ldots + b_{nb}(x)q^{-nb} .$$  \hspace{1cm} (3)

This model is referred to as ARX($na, nb$). The parameters $a_i(x), i = 1, \ldots, na$, and $b_i(x), i = 0, \ldots, nb$, are unknown (continuous) functions of the parameter $x$ and are to be estimated.

A two step identification approach can then be performed.

First: A set of $n$ different values of $x$, \{ $x_1, x_2, \ldots, x_n$ \} is chosen. Forcing $x$ to assume each one of the $n$ values, $n$ input-output sets of data are collected. Consequently, $n$ different LTI ARX models with the same structure are identified.

Second: The $a_i(x), i = 1, \ldots, na$, and $b_i(x), i = 0, \ldots, nb$, parameter functions of the LPV model are derived interpolating the corresponding parameters of the $n$ LTI models. Suitable interpolation can be performed using splines or polynomials.

With the resulting LPV model forecast and control of the output $y(k; x)$ can then be performed for any value of $x$, even different from those used in the identification stage.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{LPV approximating model for single input distributed parameter systems.}
\end{figure}

3 LPV APPROXIMATION: A SIMULATED CASE

In this section we present the simple problem of non-steady conduction in one space dimension. We consider the case of a one dimensional aluminium bar of length $l = 100cm$ plunged at its extremes into two separated fluids at different temperatures. We assume the temperature $y(t; x), x \in [0, l]$, to be the output and the temperature $T_{F1}(t)$ of the first fluid to be the input. The temperature $T_{F2}(t)$ of the second fluid is assumed to be constant.
Let the body be initially at a uniform temperature $T_0$. The general conduction equation, for uniform and isotropic materials and no internal heat generation, is driven by the following partial derivative equation (see e.g. Chapman [1984])

$$\frac{1}{\alpha} \frac{\partial T(t; x)}{\partial t} = \frac{\partial^2 T(t; x)}{\partial x^2}$$  \hspace{1cm} (4)

that is associated with the following initial and boundary conditions

$$T(0; x) = T_0, \hspace{0.5cm} \forall x \in [0, l]$$

$$-k \left( \frac{\partial T(t; x)}{\partial x} \right)_{x=0} = h \left( T(t; x) - T_{F1}(t) \right)_{x=0} \hspace{1cm} \forall t > 0$$

$$-k \left( \frac{\partial T(t; x)}{\partial x} \right)_{x=l} = h \left( T(t; x) - T_{F2}(t) \right)_{x=l} \hspace{1cm} \forall t > 0$$

where $k = 204 W/m \cdot K$ is the thermal conductivity of aluminium, $h = 0.562 W/m^2 \cdot K$ is the thermal conductance of the two fluids and $\alpha = 8.41810^{-5} m^2/s$ is the thermal diffusivity.

When a suitable input (temperature of the first fluid) $T_{F1}(t)$ is applied to the system, the simulation of the system of relation (4) allows to derive the temperature output $T(t; x)$ in any section $x$ of the bar. Such simulation has been performed in this paper by numerical integration using the implicit scheme described in Sewell [1988] and Jaluria and Torrace [1986].

If a specific section at $x = x^*$ is considered, the input-output behavior can be approximated with a time invariant lumped parameter system derived with any suitable criterion (see e.g. Ljung [1987]).

The derived LTI model accounts quite well for the temperature in $x^*$ also for input signals different from the one used for identification. Since however the goal is to monitor/control the temperature in any section of the bar, the obtained results do not give any information about the temperature in any other section different from $x^*$.

To represent with a simplified model the considered distributed parameter process still preserving some information about the spatial structure of the system, a LPV-ARX model is then constructed.

It is assumed that the process can be well represented by a LPV-ARX(3,3) model with delay whose seven parameters $a_i(x), b_i(x), i = 1, \ldots, 3$ and the delay $\Delta(x)$ are functions of the $x$ spatial variable representing the position of the section in which the output is located. The functions relating the parameters to the $x$ variable are assumed to be cubic splines to be identified from the parameters of four ARX(3,3) LTI models obtained from (2) when $x$ is forced to the four values \{ 10cm, 30cm, 50cm, 70cm \}.

Using the resulting LPV-ARX(3,3) model the temperatures in other three sections $x \{ 20cm, 40cm, 60cm \}$ not used for identification are then computed when the input is a square wave with non constant period. Such results are reported in Figure 2 (dotted lines) together with the temperatures $T(t; x)$ computed in the same sections integrating the partial differential equation (4) (solid line). As it can be seen the approximation is quite good and supports the use of such LPV models for keeping spatial information. In fact with the LPV model, considering the whole range of parameter variations, it is possible to monitor the temperature in all the sections of the bar or to design a robust control that works for all the possible parameter values.

4 LPV APPROXIMATION: A REAL CASE

In this section, we present an agricultural application that is quite close to the previously
simulated process. In intensive agriculture the continuous exploitation of the soil with the same crop leads to the development of weed seeds, nematodes and plant pathogens. It is, therefore, necessary to periodically treat the soil in order to kill these undesired agents [Mulder, 1979].

Steam heating treatments of the soil are a viable alternative to methyl bromide (CH$_3$Br), a fumigant used to treat the soil at this purpose, that will be no more employed because of its toxicity and its contribution to the Earth's ozone layer depletion. The most diffused technique to apply steam is sheet steaming which consists of covering the soil with a thermo-resistant sheet sealed at the edges and then blowing the steam under the sheet, leaving it to penetrate through the soil [Runia, 2000].

Temperature monitoring and control is then needed for soil disinfestation. The measurement data used in this paper are relative to the temperature at different depths in the soil. The data have been collected in open field after the surface of the soil was heated by steam. The interested reader should refer to Berruto et al. [2001] and Dabbene et al. [2002]. Since the conditions on the soil surface can be assumed to be uniform and the system can be macroscopically approximated with an unbounded uniform body, the only dimension that affects the system behavior is the depth $h$ at which data are collected that is the depth at which the output is located.

Temperatures were collected at depths $h = 1cm, 6cm, 11cm, 16cm$. The corresponding data are reported in Figure 3. In this case an LPV-ARX(4,4) was considered for representing the data. The parameter dependence on the depth $h$ was assumed once more to be represented by cubic splines and such functions have been derived from the data relative to $h = 1cm, 11cm, 16cm$. The LPV-ARX(4,4) parameter values for $h = 6cm$ have been computed and the model simulated. The results of the simulation together with the measurements at the same depth are reported in Figure 4. As it can be seen, also in this case the LPV model well approximates the forced response of the distributed parameter system.

![Figure 3: Soil temperatures at different depths measured during cooling.](image)

![Figure 4: LPV approximation for single input distributed parameter systems.](image)

5 CONCLUSIONS

In this paper the use of Linear Parameter Varying (LPV) models was introduced to approximate systems that are intrinsically distributed parameter processes.

The use of LPV models allows to approximate the system with a simple low order model that however still keeps some information about the spatial structure of the original system. In fact, the parameters of the LPV model are assumed to be functions of the relevant spatial coordinates that account for the distributed parameter nature of the real process. In this way the model, that remains intrinsically an input-output model, can be used for monitoring and control solutions that take into account the whole relevant range of the spatial coordinates of the real process.

This kind of approach can be quite valuable in environmental and agricultural applications where distributed parameter processes such as
those related to the diffusion of substances and/or to the transmission of heat are quite common.

It must also be noted that in these fields the collection of data in several locations can be quite expensive. It is therefore convenient to have optimal identification procedures that allow to derive the system parameters with the smallest possible uncertainty still using a moderate amount of measurements. Such procedures are available for LPV models, and for some particular cases can be even analytically derived as discussed in Belforte and Gay [2000].

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