Rheological Behaviour of Volcanic Granular Flows

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Abstract
Numerical simulations of volcanic granular flows are increasingly being used for hazard assessment on volcanoes and appear to be essential for future hazard mitigation. A potential problem of such an approach, however, is that the rheological behaviour of such flows is very complex and currently impossible to describe fully from a physical point of view. As we cannot, at present, simulate all the complexity of the interactions at a microscopic level, we have to use simplified rheological laws. The frictional behaviour is often considered as a model that well describes the first order behaviour of such flows. The aim of this paper is to compare numerical simulations to well constrained field examples to check the validity of this law. Results show that the frictional model appears to be too simple to be used for pyroclastic flow and long runout avalanche simulations. A simple empirical law, using a constant retarding stress, represents a better alternative.

Keywords: Volcanic granular flow; Debris avalanche; Pyroclastic flow; Rheology.

1. INTRODUCTION
Long runout avalanches and dense pyroclastic flows are common in volcanic environments. In the following, they are called volcanic granular flows as they are formed of blocks and ash that interact during transport. Complex interactions also exist between particles and magmatic and atmospheric gases.

Volcanic long runout avalanches form when the flank of a volcano collapses. This destabilization may result from volcanic activity (e.g. magma injection), seismic activity or weakening of the rocks through time (e.g. by hydrothermal alteration). Huge masses of rocks (from a few km$^3$ to tens of km$^3$) are then transported away (up to tens of km) at inferred velocities of about 100 m.s$^{-1}$. They form widespread deposits, extending over several tens of kilometres squared and tens of metres thick. Their granulometry commonly ranges from micrometric ashes to decametre or kilometre-sized blocks.

Dense pyroclastic flows are fast-moving bodies of hot gas, ash and lava fragments. They are generated by the gravitational collapse of lava domes or eruptive columns, or by explosions from the volcanic conduit. Dense pyroclastic flows are generally a few metres thick, have a typical volume of $10^5$-$10^8$ m$^3$ and their granulometry commonly ranges from micrometric ashes to centimetric-decimetric lapillis and blocks. Their velocity is of the order of 30-50 m s$^{-1}$. 
Both phenomena are composed of broken rocks, present a very long runout compared to their drop height, and are thin relative to their area. Existing models of dense granular flows generally start from the assumption that the frictional (i.e. Mohr-Coulomb) behaviour is dominant in their rheology and plays the major role in deposit formation (see for example Pudasaini and Hutter, 2006, and references therein). This behaviour is chosen because it is exhibited by volcanic granular deposits and it also approximates the behaviour of sand flows in the laboratory. However, before using any model in hazard assessment, we should ensure that it correctly captures the first order features of the natural phenomenon. Thus, the following key questions need to be addressed: 1) is it realistic to consider the complex rheology of volcanic granular flows as mainly frictional? and 2) is this behaviour compatible with field observations of the geometry of their deposits, their relatively low velocity and their high mobility? To answer these questions, numerical simulations are compared to well-constrained field examples.

2. VOLCFLOW

Simulations have been done with the code VolcFlow developed at the Laboratoire Magmas et Volcans. It is based on the depth-average approximation. Using a topography-linked co-ordinate system, with \(x\) and \(y\) parallel to the local ground surface and \(h\) vertical, the general depth-averaged equations of mass (eq. 1) and momentum (eqs. 2, 3) conservation are:

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0
\]

\[
\frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2)}{\partial x} + \frac{\partial (hu v)}{\partial y} = g h \sin \alpha_x - \frac{1}{2} k_{\text{actpass}} \frac{\partial}{\partial x} (gh^2 \cos \alpha_x) + \frac{T_x}{\rho}
\]

\[
\frac{\partial (hv)}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (hv^2)}{\partial y} = g h \sin \alpha_y - \frac{1}{2} k_{\text{actpass}} \frac{\partial}{\partial y} (gh^2 \cos \alpha_y) + \frac{T_y}{\rho}
\]

where \(h\) is flow thickness, \(u = (u, v)\) is flow velocity, \(\alpha\) is ground slope, \(T\) is retarding stress, \(\rho\) is the bulk density of the pyroclastic flow, \(k_{\text{actpass}}\) is the earth pressure coefficient [Savage and Hutter, 1991], and the subscripts denote components in the \(x\) and \(y\) directions. The model allows simulations of various rheologies, including frictional (with a basal and an internal friction angle), Bingham, viscous and Voelmy. More complex rheological laws can also be defined by the user.

The equations were solved using a shock-capturing numerical method based on a double upwind Eulerian scheme [Kelfoun and Druitt, 2005]. The scheme can handle shocks, rarefaction waves and granular jumps, and is stable even on complex topography and on both numerically ‘wet’ and ‘dry’ surfaces.

The value of \(k_{\text{actpass}}\) (eqs. 2 and 3) is a function of the basal (between the avalanche and the ground surface) and the internal friction angle \(\phi_{\text{bed}}\) and \(\phi_{\text{int}}\) respectively [Iverson and Denlinger, 2001].

For a dry frictional (i.e. Mohr-Coulomb) material, the retarding stress is of the form:

\[
T_x = -\rho h \left( g \cos \alpha + \frac{u^2}{r} \right) \tan \phi_{\text{bed}} \frac{u_x}{|u|}
\]

The term \(u^2/r\) takes account of “overweight” due to the centrifugal acceleration on the topography curvature [Savage and Hutter, 1991]. The term \(-\rho h u^2/|u|\) allows the \(x\)-component of the retarding stress in the direction opposed to the displacement to be calculated.

Details and tests of the scheme are developed in more detail in Kelfoun and Druitt [2005]. Some additional details on the code, other examples and downloadable files are shown on: http://wwwobs.univ-bpclermont.fr/lmv/pperm/kelfoun_k/VolcFlow.html
3. EXAMPLE OF SOCOMPA LONG RUNOUT AVALANCHE

Socompa avalanche in northern Chile has been described in papers by Francis et al. [1985], Wadge et al. [1995] and Van wyk de Vries et al. [2001]. It formed by gravitational collapse of the northwestern flank of the 6000-m-high stratovolcano, leaving an amphitheatre 12 km wide at its mouth and with cliffs 300-400 m high. The avalanche flowed across a broad topographic basin northwest of the volcano up to a maximum distance of 40 km, and covered 500 km². The volume of rock transported is estimated to be about 25 km³, with another 11 km³ preserved as intact (‘Toreva’) blocks up to 400 m high at the foot of the collapse scarp. The morphology of the avalanche deposit is perfectly preserved in the hyper-arid climate of the Atacama Desert.

3.1 Frictional behaviour

Models were run assuming a frictional avalanche rheology (eq. 4) with three different combinations of basal and internal angles of dynamic friction: (1) $\phi_{\text{bed}} << \phi_{\text{int}} = 30^\circ$, the static angle of friction for dry granular debris; (2) $\phi_{\text{bed}} \neq 0^\circ$ but $\phi_{\text{int}} = 0^\circ$; (3) $\phi_{\text{bed}} = \phi_{\text{int}} \neq 0^\circ$ [Kelfoun and Druitt, 2005].

All three of these frictional models reproduce only very crudely the shape of the real avalanche deposit. A major failing is that, owing to the very low basal friction, the model avalanches flow off any gradients greater than 1 to 2.5° (depending on the case). After reaching their maximum limits, the avalanches drain back into the centre of the Monturaqui Basin. Consequently, the model deposits have negligible thickness along their limits of maximum extent, whereas thicknesses of up to 60 m are observed along the margins of the actual avalanche [Wadge et al., 1995]. The effect of topographic draining is to cause excess concentration of debris on the floor of the Monturaqui Basin.

![Figure 1. Snapshots of the emplacement of frictional avalanche models 1 and 2 at t = 200 s and t = 400 s, with the corresponding deposits (c and g) from Kelfoun and Druitt, 2005. (a-d) Model 1. Avalanche with $\phi_{\text{bed}} = 1^\circ$ and $\phi_{\text{int}} = 30^\circ$. (e-h) Model 2. Avalanche with $\phi_{\text{bed}} = 2.5^\circ$ and $\phi_{\text{int}} = 0^\circ$. The colour scale gives the thicknesses (m) of the avalanche. Figures d and h are shaded relief maps of the final deposits.](image-url)
Simple frictional models are able to reproduce the approximate runout of Socompa avalanche only if very low values are used for the basal dynamic friction. However, they are unable to generate deposits with realistic thicknesses on slopes greater than about three degrees, nor realistic surface morphology.

### 3.2 Constant retarding stress

In view of the apparent inadequacy of the simple frictional models, Kelfoun and Druitt [2005] also ran models in which the retarding stress $T$ in equations 2-3 was constant ($k_{\text{actpass}}$ was taken as unity). This very simple assumption was motivated by the study of Dade and Huppert [1998], who found that the field data for a large number of avalanches can be explained by an approximately constant retarding stress.

This empirical law simply states that the retarding stress is constant, independent of the velocity, thickness or any other parameter of the flow:

$$ T = \text{Constant} \times \frac{\mu}{|\rho|} \quad \text{or} \quad T = \left\| T_x + T_y \right\| = \text{Constant} \quad [6] $$

The models produce surprisingly good fits to the real avalanche, provided that $T$ is about 50 kPa, depending on the initial slide geometry chosen. Unlike the frictional rheologies, this law produces a deposit with a well defined edge and leaves a deposit of realistic thickness on all slopes, irrespective of angle. Surface structures on the deposit from this simulation are remarkably similar to those of the real avalanche (Fig. 2).

![Figure 2](image)

**Figure 2.** Avalanche evolution using a constant retarding stress $T = 52$ kPa [from Kelfoun and Druitt, 2005]. The colour scale denotes thickness. (a-c) Snapshots at 200 s, 400 s and 600 s. (d) Shaded relief map of the simulated deposit. (e) Shaded relief map of the real deposit.

A recent study of Kelfoun et al. [2008], based on field work and imagery analysis confirm the movement observed in the numerical model of Kelfoun and Druitt [2005]. It thus proves that the constant retarding stress is a good estimation of rheology of the avalanche.
4. EXAMPLE OF TUNGURAHUA PYROCLASTIC FLOWS

Tungurahua volcano in Ecuador erupted in July and August 2006, emitting pyroclastic flows that swept along the west flank of the volcano. The eruption was observed, described and monitored by the staff of the Instituto de Geofísico of the Escuela Politécnica Nacional (IG-EPN), Quito. Based on their data, it is possible to reconstruct the conditions of pyroclastic flow formation, transport and emplacement.

Results of this section are developed in Kelfoun et al. (submitted). The thickness of each unit is about 1 m, the total thickness of the deposit, composed of several units, being generally less than 10 metres. The feeding duration of the paroxysmal phase was 40 minutes and the total volume emitted between 5 and \(10 \times 10^6\) m\(^3\). Pyroclastic flows were formed in a quasi continuous manner close to the crater, with no, or very low, initial velocity. The maximal elevation at which pyroclastic flows are generated is 5000 m, the eastern rim is unbreached by pyroclastic flows. Field observations indicate that a greater volume of pyroclastic flows formed at the low points of the rim than at higher, and it is assumed that the mass rate at the feeding cells, expressed by the variation of the thickness \(dh\) with time \(t\), is a linear function of the difference between the maximal elevation \(z_{gen} = 5000\) m where pyroclastic flows were generated and \(z_{rim,i}\), the elevation of the crater rim.

![Figure 3. Map of deposits, Tungurahua, august 2006. Dense pyroclastic flows are in brown (from Kelfoun et al., submitted).](image)

The new mass \(dh\) is assumed to be poured without velocity, the velocity of the cells at the feeding points being calculated by the conservation of momentum. In the simulation, the source is assumed as being at the cells located just outside the crater rim, at a distance of 350 m from the crater centre.

Frictional behavior

The frictional model that best reproduces the extension of the August 2006 eruptions of Tungurahua assumes \(\phi_{bed} = 15^\circ\) for \(\phi_{int}\) equaling 0. The value of \(\phi_{int}\) modifies runout and velocity of the flows but does not significantly change the conclusions of the paper (as for Socompa) and is not detailed here. At the onset of the eruption, the mass poured on the crater rim immediately began to flow and rapidly covered all the SW, W and NW flanks with a very thin thickness of <0.1 mm. The main drainage channels were affected, but also interfluves. The mass accelerated along the steep (>30\(^\circ\)) slope of the terminal cone, reaching a velocity of more than 200 m.s\(^{-1}\). The thickness of pyroclastic flows on the interfluves does not significantly increase with time, but it increased in the valleys into which the mass drained. The mass began to decelerate and accumulate where the slope is inferior to the friction angle. After the alimentation stopped, at \(t = 2400\) s, all the mass deposited on the flank progressively drained down and accumulated at the foot of the volcano on slopes slightly lower than the friction angle, where it formed sand piles with surface angle of about 15\(^\circ\) (Fig.4b).
The incompatibility between numerical results and field data is not due to a problem in the numerical code or DEM resolution: put a grain of frictional material (sand for example) at the surface of a curved slope and the grain will immediately slide along the slope and will stop a little bit after the slope becomes less than the friction angle of the material. Put several grains and they will all stop at approximately the same place. Pour the sand over some tens of seconds and the sand will flow and accumulate to form a sand pile. Our frictional results reproduce exactly this frictional behaviour.

These results could point out a potential problem of using the frictional model for hazard assessment. One could argue that the shape of deposits is not important for hazard assessment and that a good map of inundation can be obtained with a simple model that does not capture all the complexity of the physics of the phenomenon. But results show that the frictional model cannot reproduce a correct map of inundation at Tungurahua. Firstly, all the interfluves are affected by dense flows and this largely overestimates the areas which are threatened (interfluves may have been affected by surges but the frictional model is not adapted for their simulation). Secondly, the mass accumulates in thick piles (up to 50m) and cannot spread. The model thus underestimates the hazard in flat areas. Thirdly, a velocity 5 to 10 times higher than in reality allows pyroclastic flows to cross ridges that they would not be able to cross in reality. The portion of the flows that crosses the ridge is no longer available to follow the drainage and to affect downstream areas. In this case of drainage bending, the frictional model can greatly underestimate areas affected. In the case of the Tungurahua, this occurs to the south: the deposits formed by the model at \( \gamma = 9.833 \times 10^6 \) (Fig. 4) crossed a 100 m-high ridge. No deposits were found here in the field; instead the mass followed the main river to affect the populated area of Palitahuia (Fig. 3).

It should also be noted that all the characteristics described here which show that frictional behaviour differs from pyroclastic flows are largely independent of the total volume, feeding conditions, details of the topography, value of the friction angle chosen, etc. The same behaviour occurs when the rate, volume, or the way the mass is injected are changed.

**Constant retarding stress**

The best fit results (extension, thickness, velocity) for the constant retarding stress are obtained with a constant retarding stress of 7kPa (Fig. 5).
The first difference with the frictional model is that the mass accumulates around the rim before reaching a certain thickness (about 1 m) that allows flow initiation. Figure 5a shows that emplacement is carried out by pulses (in red) even if the source rate is constant: the mass accumulates until it reaches a critical thickness (about 1 m) which enables flow to take place. The mass then continues to be accumulated until the critical thickness is reached again, and a new pulse forms. Pulses are thicker and faster than the neighbouring material. Pulses generally run at less than 30m.s⁻¹, which corresponds to field observations. The front of the flows is well defined during emplacement. The simulated deposits also show a well defined front and a progressive decrease in thickness on steeper slopes. Simulated dense pyroclastic flows present a realistic thickness (0.5 – 2 m thick) and are restricted to valleys (Fig 5).

**Figure 5:** Velocity of pyroclastic flows at t=2400s (a) and thickness of deposits (b), simulated using the constant stress model (7 kPa). The colour scale of the thickness (b) is logarithmic. Black contours indicate 0.1 m deposits. From Kelfoun et al., submitted.

**OTHER EXAMPLES**

Socompa and Tungurahua are two examples among other field cases that have also been tested including pyroclastic flows on Lascar (Chile) and Merapi (Indonesia), and avalanches on Lastarria (Chile) and Llullaillaco (Argentina). Constant stress is always a better rheology than frictional stress. The value of the constant retarding stress is less than 10 kPa for pyroclastic and pumice flows, and about 50 kPa for long runout avalanches.

**CONCLUSIONS**

The frictional model appears not to be the best adapted to simulate pyroclastic flow and debris avalanche emplacement and deposits at Tungurahua and Socompa, as well as at other field example studied. Contact between particles at rest is surely frictional but other phenomena act when particles are moving to give a much more complex behaviour to the flow. The constant retarding model appears much better to capture this complex behaviour and to simulate pyroclastic flows.

The greater adequacy of the constant retarding stress proves that the ratio of driving / retarding stress in granular flows cannot be considered as constant – as for the frictional model – but decreases as the thickness increases. Above a certain thickness the flow is able to move. Less than this thickness, the retarding stress dominates the driving stress and the flow slows down. This produces flow deposits with a more or less constant thickness that
decreases on slopes, and this appears consistent with field observations. The behaviour is probably explained by an increase of the mechanical strength of the flows from their base or interior to the surface. In Socompa avalanche, a detailed study of the deformation of the surface lithology shows that the avalanche was formed by a fluidal interior covered by a rafted brittle crust (Kelfoun et al., 2008). This change in mechanical properties may be related to the variation of granulometry from bottom to top: the interior of the flow being constituted of finer particles than the surface, it may, for example, retain gas more easily thus prolonging its flowing capability. This increase of granulometry towards the surface, common in granular flow deposits, was observed at Socompa, and also appears very clearly in the Tungurahua and Lascar deposits.

The physics of pyroclastic flows is complex and many developments are necessary to fully understand it, and to obtain a robust physical model. Results of this paper will be useful for future evaluation of the quality of such a complex model that will have to explain why a constant stress model appears much more adapted than a frictional model to simulating pyroclastic flows. In the meantime, the frictional model appears to be too simple to be used for pyroclastic flow and long runout avalanche simulations. Constant retarding stress seems to represent an acceptable alternative.

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